UNIVERSITY OF TORONTO

ERINDALE CAMPUS

SURVEY SCIENCE

AN ALGORITHM FOR THE AZIMUTH OF POLARIS

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During the past year Survey Science at Erindale College has had numerous requests for programs to compute the azimuth of the sun or Polaris without recourse to astronomical tables. For those who desire to produce their own Solar Azimuth program we suggest G.G. Bennett's algorithm given in his article, "A Solar Ephemeris for use with Programable Calculators", published in the September, 1980 issue of the Australian Surveyor and reprinted in "North Point", or the algorithm given by M.R. Craymer in the Survey Science Technical Report No. 3, Erindale College.

The azimuth of Polaris may be calculated using the following algorithm which has been designed to give a calculation accuracy of one second of arc. This algorithm is sufficiently concise that it can be used to program a large capacity calculator for field use or a small microcomputer. Alphanumeric notation is used throughout the algorithm for ease in programming. The numerical example is provided to assist in debugging the program. A word of caution -- some calculators do not give accurate trignometric functions for large angles so that it may be necessary to reduce all angles obtained from the equations to values less than 360 degrees or the equivalent in radians.

For those who wish to delve further into the formulation, the following references may prove useful.

Craymer, M.R. Azimuth Determination from Observations on Polaris and the Sun, Survey Science Technical Report No. 3, Erindale College, 1984.

Meeus, Jean. Astronomical Formulae for Calculators, 2nd ed. William - Bell, Inc., Richmond, Va., 1982. (available from Astronomy or Sky and Telescope magazines)

McNally, D. Positional Astronomy, Frederick Muller Ltd., London, 1974.

Mueller, I. Spherical and Practical Astronomy as Applied to Geodesy, New York, Ungar, 1969.

Smart, W.M. Spherical Astronomy, 4th edition, Cambridge University Press, 1960.

POLARIS AZIMUTH ALGORITHM

Gunn and Craymer

1985

Julian Date

JD = INT(365.25 Y) + INT(30.6001(M+1)) + DY + UT/24 + 1720994.5 + B

where

Y = year (4 digit)

M = month

DY = day

UT = universal time in hours

A = INT(Y/100)

B = 2 - A + INT(A/4)

and if M $\leq$ 2, Y = Y - 1

M = M + 12

Note that INT indicates the truncated integer value (i.e., INT (3.99) = 3)

Ephemeris and Tropical Centuries

TE = (JD - 2415020.0) / 36525 (Julian ephemeris centuries)

T = (JD - 2442413.478) / 36524.2199 (tropical centuries)

Orbital Parameters

$$M = 358.475833 + 35999.04975\ TE - 0.00015\ TE^2 - 0.000003\ TE^3\ \text{(deg)}$$

$$D = 350.737486 + 445267.114217\ TE - 0.001436\ TE^2\ \text{(deg)}$$

$$F = 11.250889 + 483202.02515\ TE - 0.003211\ TE^2\ \text{(deg)}$$

$$G = 259.183275 - 1934.142008\ TE + 0.002078\ TE^2\ \text{(deg)}$$

Longitude of the Sun

$$L = 279.696678 + 36000.768925\ TE + 0.000303\ TE^2$$
$$+ (1.9194603 - 0.0047889\ TE - 0.0000144\ TE^2)\ \sin M$$
$$+ (0.0200939 - 0.0001003\ TE)\ \sin 2M$$
$$+ 0.0002928\ \sin 3M + 0.0000050\ \sin 4M\ \text{(deg)}$$

FK4 Catalogued 1975.0 Data for Polaris

AC = 31.8605670 (deg)

UA = 0.0853410 (deg/century)

DA = 0.0423915 (deg/century^2)

DC = 89.1499556 (deg)

UD = -0.0002167 (deg/century)

DD = -0.0004389 (deg/century^2)

Proper Motion Correction

$$A1 = AC + UA*T + 0.5*DA*T^2$$

$$D1 = DC + UD*T + 0.5*DD*T^2$$

General Precession Correction

$$C = (2305.297\ T + 0.302\ T^2)/3600\ \text{(deg)}$$

$$Z = (C + 0.791\ T^2)/3600\ \text{(deg)}$$

$$R = (2004.042\ T - 0.426\ T^2)/3600\ \text{(deg)}$$

$$A2 = \arctan\left(\frac{\cos D1\ \sin(A1 + C)}{\cos R\ \cos D1\ \cos(A1 + C) - \sin R\ \sin D1}\right) + Z$$

$$D2 = \arcsin(\sin R\ \cos D1\ \cos(A1 + C) + \cos R\ \sin D1)$$

Nutation Parameters

$$NL = [-17.233\ \text{Sin}G + 0.209\ \sin 2G - 1.273\ \sin(2G-2D+2F) + 0.126\ \sin M - 0.204\ \sin(2G+2F)]/3600\ \text{(deg)}$$

$$NE = [9.210\ \cos G - 0.090\ \cos 2G + 0.552\ \cos(2G-2D+2F) + 0.088\ \cos(2G+2F)]/3600\ \text{(deg)}$$

Mean Obliquity of the Ecliptic

$$EM = 23.452294 - 0.013013\ TE - 0.000002\ TE^2\ \text{(deg)}$$

Apparent obliquity of the Ecliptic

E = EM + NE

Nutation In Right Ascension and Declination

DA1 = -NE cosA2 tanD2 + NL(cosE + sinE sinA2 tanD2)

DD1 = NE sinA2 + NL sinE cosA2

A3 = A2 + DA1

D3 = D2 + DD1

Annual Circular Aberration Correction (Apparent Right Ascension and Declination)

DA2 = -20.496 secD3 (cosL cosE cosA3 + sinL sinA3) (arcsec)

DD2 = -20.496 [cosL cosE(tanE cosD3 - sinD3 sinA3) + cosA3 sinD3 sinL] (arcsec)

A = A3 + DA2 (Apparent Right Ascension)

D = D3 + DD2 (Apparent Declination)

Greenwich Mean Sidereal Time

GMST = UT + 6.64606556 + 2400.051262 TE + 0.000026 TE^2 (hr)

Where UT is the Universal Time of observation in hours

Greenwich Apparent Sidereal Time:

GAST = GMST + NL cosE

where NL must be in hours

Local Hour Angle

$H = GAST - A - LW$

where

LW = West longitude of the observer in degrees and

GAST must be in degrees.

Azimuth

$$AZ = \arctan\left(\frac{\sin H}{\sin P \cos H - \cos P \tan D}\right)$$

where

P = latitude of the observer

If $AZ < 0$ then $AZ = AZ + 360°$

EXAMPLE SOLUTION

August 10, 1985

Universal Time $2^h 13^m 10^s$

Latitude 45°40'32"

Longitude 80°10'15"

Julian Date

DY = 10   M = 8   Y = 1985

A = 19   B = -13

JD = 2446287.593

Ephemeris and Tropical Centuries

TE = 0.856060041

T = 0.106069754

Orbital Parameters

M = 31175.82373 (deg)

D = 381526.1205 (deg)

F = 413661.1941 (deg)

G = -1396.556889 (deg)

L = 31097.41429 (deg)

A1 = 31.86985757 (deg)

D1 = 89.14993015 (deg)

General Precession

C = 0.06792222 (deg) Z = 0.06792500 (deg)

R = 0.05904444 (deg)

A2 = 34.24155243 (deg)

D2 = 89.19942812 (deg)

Nutation

NL = -0.00294722 (deg) NE = 0.00185833 (deg)

Obliquity of the Ecliptic

EM = 23.44115262 (deg)

E = 23.44301206 (deg)

Nutation in Right Ascension and Declination

DA1 = -0.159920708 (deg)

DD1 = +0.000077031 (deg)

A3 = 34.08163172 (deg)

D3 = 89.19950515 (deg)

Annual Aberration

DA2 = +0.073462453 (deg)

DD2 = -0.005322171 (deg)

A = 34.15509417 (deg)

D = 89.19418298 (deg)

Greenwich Sidereal Time

GMST = 2063.453511 (hr)

GAST = 2063.453331 (hr)

Local Hour Angle

H = 30837.47405 (deg)

= 237.47405 (deg)

Azimuth of Polaris

AZ = 0.964882690 (deg)

$(0^\circ 57' 53''.6)$