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AZIMUTH DETERMINATION FROM OBSERVATIONS ON POLARIS AND THE SUN

by

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ADDENUM

The following corrections should be made to the text on the indicated pages.

Page	
27	$w = 101.220833 + \ldots$
33	$d(\log R) = d(\log R)_{mn} + d(\log R)_{mc} + d(\log R)_{v} + d(\log R)_{m} + d(\log R)_{j} + d(\log R)_{sn}$
39	$\alpha_o' = \alpha_o + u(t-t_o) + 0.5\ du/dt\ (t-t_o)^2$
	$\delta_o' = \delta_o + u'(t-t_o) + 0.5\ du'/dt\ (t-t_o)^2$

ABSTRACT

As a result of the growing popularity of microcomputers and programmable calculators with large storage capacities, it is now feasible to directly compute the azimuth of the Sun and Polaris given only the time of observation and the observer's astronomic latitude and longitude. The methods needed to calculate the Sun's coordinates are based upon the same theory of motion of the Earth around the Sun that is presently used to produce The Astronomical Almanac (prepared jointly by the United States and British Nautical Almanac Offices). The popular ephemerides used by most land surveyors (i.e. The Star Almanac and the K&E Solar Ephemeris) Canadian Ministry of Energy, Mines and Resources are compiled from the fundamental ephemeris. The major purpose of this report is to present the expressions necessary to compute the Sun's astronomical coordinates to the same precision currently available in the fundamental ephemeris. The conventional method of updating Polaris' coordinates is also outlined for completeness. A computer program incorporating these expressions in the determination of astronomical azimuth from observations on the Sun or Polaris is provided in the Appendix.

ACKNOWLEDGEMENTS

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1. INTRODUCTION

The fundamental ephemerides of the Sun (The Astronomical Almanac - formerly The Astronomical Ephemeris or The American Ephemeris) have, until recently, been the only source of positional coordinates available to most surveyors. Attempts have been made to simplify astronomical data using simple polynomials (e.g. Sinclair [1975], The Star Almanac and the Almanac for Computers), Chebyshev polynomials (e.g. the Almanac for Computers) and Fourier series (e.g. Bennett [1978]). From 1977 onwards, both the United States and British Nautical Almanac Offices began supplying polynomial coefficients; the former in the Almanac for Computers and the latter in The Star Almanac.

Except when two or more sets of data are required during a short interval of time, the polynomial expressions can be as inconvenient and cumbersome as interpolation from tables. To retain the required accuracy, the polynomials require many terms and/or are limited to relatively short periods of time. Table 1 compares some of the currently available sources of polynomial coefficients with respect to the number of terms, span of validity and maximum error.
It can be seen from this Table that the polynomials may not be any more convenient than interpolation from tables. From the point of view of high precision, the large number of terms (as many as 36 for Greenwich Apparent Sidereal Time) create as much work and more potential sources of error when

TABLE 1

Comparisons of polynomial expressions

	No. of Terms	Span of Validity	Max Error
Star Almanac:			
R(GHA ♈)	2	32d	0".5
E(GHA Sun)	5	32d	1".5
Dec	5	32d	0".5
Semi-Diameter	2	32d	0".4
Almanac for Computers N.A. Series:			
GHA	6	32d	1".2
GHA Sun	6	32d	1".2
Dec	6	32d	1".2
Semi-Diameter	6	32d	0".6
Almanac for Computers A.E. Low Precision:			
GAST(0hr UT)	10	1yr	0".5
RA	22	1yr	9".0
Dec	22	1yr	3".0
Semi-Diameter	22	1yr	0".05
Almanac for Computers A.E. High Precision:			
GAST(0hr UT)	36	95d	0".02
RA	22	95d	0".3
Dec	22	95d	0".1
Semi-Diameter	22	95d	0".02

inputing all of the required coefficients. In addition, approximations in both The Star Almanac and the Almanac for Computers (NA series), containing five and six terms respectively, are limited to only a 32 day time span, resulting in impractical applications for infrequent users. Similar arguments can also be made against the use of Fourier series.

The ideal method of determining the Sun's coordinates would only require the input of the time of observation and would not be restricted to specific time periods. This criteria could be satisfied by deriving the Sun's ephemeris from the same theories of celestial mechanics upon which the fundamental ephemerides are based. However, the United States Nautical Almanac Office has rejected this idea stating [United States Nautical Almanac Office, 1979]:

> Expressions for direct calculations must take the form of mathematical approximations since the precise data contained in the ephemerides are calculated from extensive theories which are not readily adaptable to the majority of astronomical and navigational applications.

Considerable advances in the computer field now require this argument to be reconsidered. Meeus [1962] has taken the first step in this direction by compiling and deriving expressions and algorithms for many astronomical problems, including the calculation of the coordinates of the Sun. However, the precision resulting from Meeus's expressions is unacceptable to users such as land surveyors. More recently, Bennett [1980] has given a similar algorithm with greater precision.

The determination of the coordinates of Polaris (right ascension and declination) is performed in the traditional manner by updating the coordinates from one epoch (1950.0) to another. This method has been well documented by many authors (e.g. Mueller [1969]) and will therefore only be outlined briefly here for completeness.

It is the aim of this report to describe the expressions and algorithms required for the computation of the astronomical coordinates for both the Sun and Polaris. These will provide both high precision and a length of validity limited only by significant changes in the system of astronomical constants. The expressions may then be utilized in specific computer programs for azimuth determination. An example of such a program is provided in the Appendix.

Throughout this report the units are explicitly given at the end of equations for which coefficients determine the units. Note that distances expressed in astronomical units (AU) are defined by [Stein, 1982]:

$$1 \text{ AU} = 1.49597870 \times 10^{11} \text{ metres.}$$

It should also be brought to the readers attention that the figures depicting the celestial sphere are actually distorted to aid in their construction. This distortion follows the same pattern as many introductory texts in positional astronomy (e.g. Mueller [1969]).

2. COORDINATE SYSTEMS AND TRANSFORMATIONS

A brief description of geocentric coordinate systems and transformations used in this report is presented in this Chapter. Due to their great distances, celestial objects are generally considered to be projected onto a sphere of unit radius referred to as the celestial sphere. Consequently, the location of such objects may be expressed in a suitably chosen two-dimensional, curvilinear coordinate system. These coordinate systems are the Ecliptic, Right Ascension, Hour Angle and Horizon systems.

2.1 ECLIPTIC SYSTEM

The coordinate axes defining this system are illustrated in Figure 1. The origin of this system is at the centre of mass of the solar system, usually considered to be at the centre of the Sun. The x-axis points in the direction of the vernal equinox (Υ - see 3.2) and the z-axis is aligned perpendicular to the ecliptic, defined as the plane containing the orbit of the Earth-Moon system around the centre of mass of the solar system [Mueller, 1969]. The y-axis is chosen so as to make the system right-handed. The North Ecliptic Pole (NEP) is located at the intersection of the z-axis with the celestial sphere and the angle of intersetion of the ecliptic and equator is called the Obliquity of the Ecliptic (ε).

The curvilinear coordinates of a point in this system are the ecliptic latitude (β) and longitude (λ). Their definitions are evident from Figure 1, where 's' represents an arbitrary point.

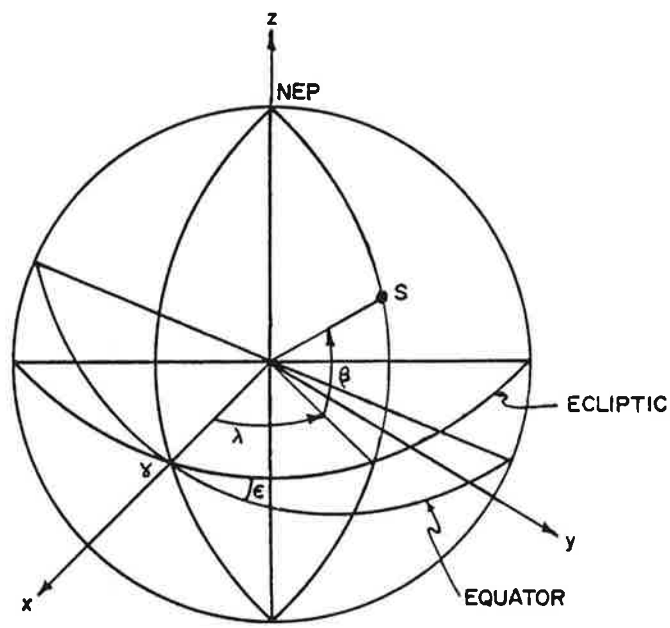

Figure 1: The ecliptic system

2.2 RIGHT ASCENSION AND HOUR ANGLE SYSTEMS

This system is illustrated in Figure 2. The origin of this system is at the centre of mass of the Earth. As for the ecliptic system, the x-axis points in the direction of the vernal equinox (Υ). The z-axis is aligned with the spin axis of the Earth and the y-axis is oriented to make the system right-handed. The North Celestial Pole (NCP) is at the intersection of the z-axis with the celestial sphere.

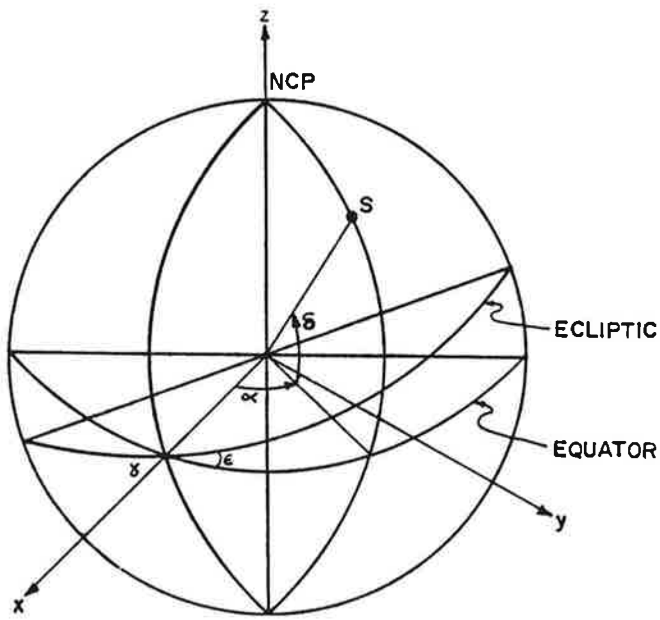

Figure 2: The right ascension system

The vernal equinox is the point of intersection of the ecliptic and the celestial equator where the apparent Sun crosses the ecliptic from south to north.

The curvilinear coordinates of a point are the right ascension (α) and declination (δ). Their definitions are apparent from Figure 2, where 's' represents an arbitrary point. Alternatively, the hour angle (h) may be used instead of the right ascension (see Figure 3). The relationship between h and α is given in 2.4.2. Note that this hour angle system is left-handed, where the x-axis points in the direction of the observer's local meridian.

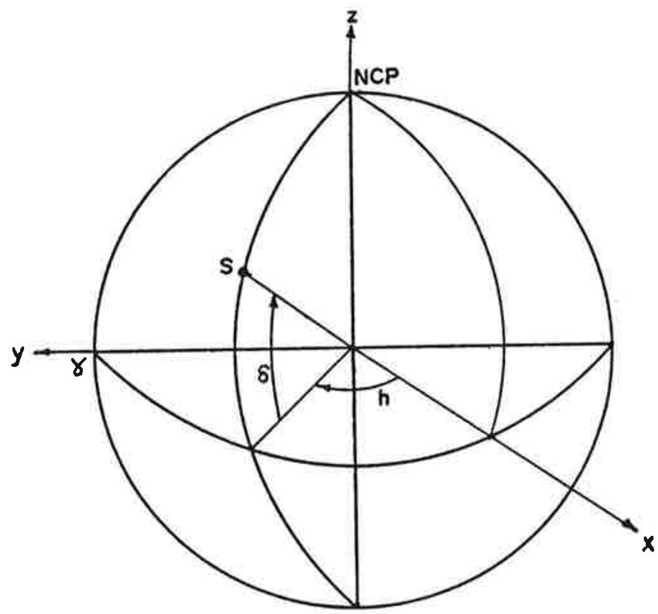

Figure 3: The hour angle system

2.3 HORIZON SYSTEM

This system is illustrated in Figure 4 where the origin is at the point of observation (i.e. on the surface of the Earth). The z-axis points in the direction of the observer's zenith (Z) and the y-axis makes the system left-handed.

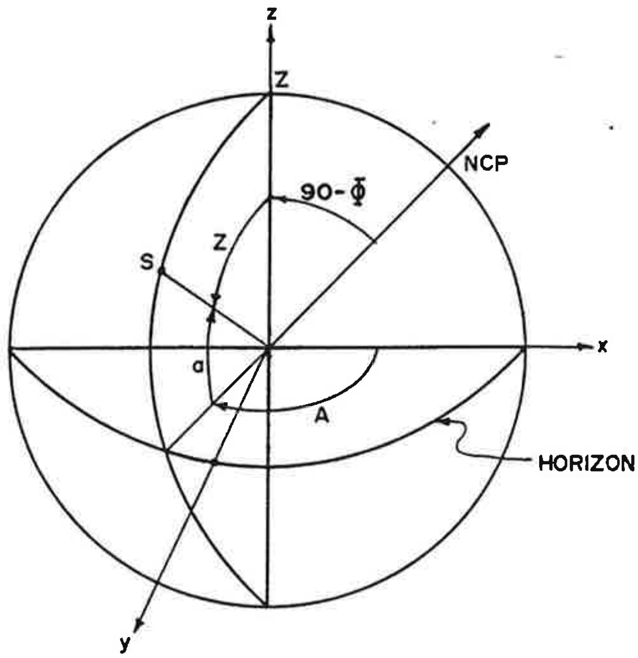

Figure 4: The horizon system

Due to the great distances involved with astronomic observations, the origin of this system is usually considered to be at the centre of mass of the Earth. The effect of this on the curvilinear coordinates is outlined in Chapter 9.

The azimuth (A) and altitude (a) or zenith distance (z) are the curvilinear coordinates as seen in Figure 4. The angle between the NCP and z-axis is the astronomic latitude (Φ) of the observer and should not be confused with the ecliptic latitude (β). The azimuth (A) and altitude (a) or zenith distance (z) define the curvilinear coordinates in this system as can be seen from Figure 4.

2.4 TRANSFORMATIONS BETWEEN SYSTEMS

In order to simplify these transformations, the coordinate systems are all considered to be geocentric. The corrections to this presumption are outlined in Chapter 9. Furthermore, matrix algebra and cartesian coordinates are employed here. The right-handed, 3 x 3, orthogonal rotation matrices about the x, y and z axes, denoted by Rx, Ry and Rz respectively, are defined by:

$$Rx(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$Ry(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$Rz(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

2.4.1 Ecliptic and Right Ascension

It can be seen that the only difference between the two systems is a rotation around the x-axis (the axis containing the vernal equinox and the origin) by an amount equal to the obliquity of the ecliptic (ε).

On the celestial sphere the cartesian coordinates of both systems may then be expressed in terms of the curvilinear coordinates as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Ecl} = \begin{bmatrix} \cos\beta \cos\lambda \\ \cos\beta \sin\lambda \\ \sin\beta \end{bmatrix} ,$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} = \begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{bmatrix} .$$

The rotation matrix, Rx, is used to rotate either system about the x-axis by the angle ε:

$$Rx(\varepsilon) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varepsilon & \sin\varepsilon \\ 0 & -\sin\varepsilon & \cos\varepsilon \end{bmatrix} .$$

The transformations between the two systems may then be performed as follows:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} = Rx(-\varepsilon) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Ecl} ,$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Ecl} = Rx(+\varepsilon) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} .$$

Inserting the curvilinear coordinates and performing the matrix multiplication, the following relationships are derived:

Ecliptic to Right Ascension

$$\begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{bmatrix} = \begin{bmatrix} \cos\beta \cos\lambda \\ \cos\beta \sin\lambda \cos\varepsilon - \sin\beta \sin\varepsilon \\ \cos\beta \sin\lambda \sin\varepsilon + \sin\beta \cos\varepsilon \end{bmatrix}$$

$$\alpha = \arctan(\tan\lambda \cos\varepsilon - \tan\beta \sin\varepsilon \sec\lambda)$$

$$\delta = \arcsin(\cos\beta \sin\lambda \sin\varepsilon + \sin\beta \cos\varepsilon) \quad ,$$

Right Ascension to Ecliptic

$$\begin{bmatrix} \cos\beta \cos\lambda \\ \cos\beta \sin\lambda \\ \sin\beta \end{bmatrix} = \begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \cos\varepsilon + \sin\delta \sin\varepsilon \\ -\cos\delta \sin\alpha \sin\varepsilon + \sin\delta \cos\varepsilon \end{bmatrix}$$

$$\lambda = \arctan(\tan\alpha \cos\varepsilon + \tan\delta \sin\varepsilon \sec\alpha)$$

$$\beta = \arcsin(-\cos\delta \sin\alpha \sin\varepsilon + \sin\delta \cos\varepsilon) \quad .$$

2.4.2 <u>Right Ascension and Horizon</u>

The relationships between these systems are normally derived through the hour angle system. In terms of the curvilinear coordinates, the cartesian coordinates may be expressed as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{HOR} = \begin{bmatrix} \cos a \cos A \\ \cos a \sin A \\ \sin a \end{bmatrix} = \begin{bmatrix} \sin z \cos A \\ \sin z \sin A \\ \cos z \end{bmatrix} \quad .$$

As seen in Figures 2 and 3 the conversion from the horizon system to the hour angle system requires first a rotation around the z-axis of 180 ^{o} and then a rotation of ($\Phi-90^{o}$) about the y-axis. The subsequent conversion to the right ascension system requires a change of handedness, Py, and a negative rotation of an amount equal to the local apparent sidereal time (LAST - see next section). In matrix notation this is given as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} = Rz(-LAST)PyRy(90^{o} - \Phi)Rz(180^{o}) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Hor}$$

and the inverse is:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Hor} = Rz(180^\circ)Ry(\Phi - 90^\circ)PyRz(LAST) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} .$$

Here, the change of handedness, Py, is given as a reflection of the y-axis:

$$Py = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

After the appropriate substitutions the following equations may be obtained:

Horizon to Hour Angle

$$\begin{bmatrix} \cos\delta \sinh \\ \cos\delta \sinh \\ \sin\delta \end{bmatrix} = \begin{bmatrix} \cos z \cos\Phi - \sin z \cos A \sin\Phi \\ -\sin z \sin A \\ \cos z \sin\Phi + \sin z \cos A \cos\Phi \end{bmatrix}$$

$$h = \arctan[\sin A / (\cos A \sin\Phi - \cos\Phi \cot z)]$$

$$\delta = \arcsin(\cos z \sin\Phi + \sin z \cos A \cos\Phi) \quad ,$$

Hour Angle to Horizon

$$\begin{bmatrix} \sin z \cos A \\ \sin z \sin A \\ \cos z \end{bmatrix} = \begin{bmatrix} \sin\delta \cos\Phi - \cos\delta \cosh \sin\Phi \\ -\cos\delta \sinh \\ \sin\delta \sin\Phi + \cos\delta \cosh \cos\Phi \end{bmatrix}$$

$$A = \arctan[-\cos\delta \sinh / (\sin\delta \cos\Phi - \cos\delta \cosh \sin\Phi)]$$

$$= \arctan[\sinh / (\cosh \sin\Phi - \cos\Phi \tan\delta)]$$

$$z = \arccos(\sin\delta \sin\Phi + \cos\delta \cosh \cos\Phi) \quad ,$$

Hour Angle to Right Ascension

$$\alpha = LAST - h \quad ,$$

Right Ascension to Hour Angle

$$h = LAST - \alpha \quad .$$

3. TIME SYSTEMS

There are two basic terms commonly used to define time: the interval and the epoch. The interval is the amount of time that has elapsed between two events. The epoch is the amount of time that has elapsed between a specific reference event, called the fundamental epoch,and the occurence of another event. Strickly speaking, the epoch may be considered as a time interval referred to a fundamental epoch.

The following systems of time are merely different time scales with which the time interval is measured. The four basic time scales in use today are based upon the following natural, observable phenomena:

1. Ephemeris Time: based on the orbital motions of the planets.
2. Sidereal Time: based on the diurnal rotation of the Earth with respect to the stars.
3. Solar Time: based on the diurnal rotation of the Earth with respect to the Sun.
4. Atomic Time: based on the electromagnetic oscillations produced by the quantum transition of an atom of Cessium 133 [Thomson, 1978].

3.1 EPHEMERIS TIME

This is the theoretically uniform time system based upon the variation of the Sun's geometric, ecliptic longitude. Ephemeris Time (ET) is the independent variable in the orbital theories of the planets and closely agrees with Universal Time (see 3.3) although no specific relationship exists between the two systems. Ephemeris Time is used as the time argument for a number of tables in the fundamental ephemerides.

Newcomb's [1898a] theory of the apparent motion of the Sun has been adopted by the I.A.U. as the basis for this system. The origin and rate of Ephemeris Time were therefore chosen so as to agree with Newcomb's expression for the mean ecliptic longitude of the Sun, L, referred to the mean equinox of date [N.A.O., 1961]:

$$L = 279.6966778 + 36000.7689250\ Te + 0.0003025\ Te^2 \ (\text{deg}) ,$$

where Te is the interval of Julian ephemeris centuries of 36525 days that have elapsed since the fundamental epoch of Ephemeris Time; 1900 January 0.5d ET.

The fundamental epoch has been more formally defined by the I.A.U. in the following manner [International Astronomical Union, 1960]:

> Ephemeris time is reckoned from the instant, near the beginning of the calendar year AD 1900, when the geometric mean longitude of the Sun was $279^\circ 41' 48''.04$ at which time the measure of ephemeris time was 1900 January 0.5d precisely.

The Julian ephemeris date (JED) corresponding to this epoch is JED 2415020.0, from which the interval of Julian ephemeris centuries, Te, elapsed from this date can be expressed in terms of the Julian ephemeris date as:

$$Te = (JED-2415020.0)/36525 \quad \text{(Julian ephemeris centuries)}$$

3.2 SIDEREAL TIME

This time system is based on the diurnal motions of the stars and is therefore a direct measure of the rotation of the Earth. The epoch of Sidereal Time, ST, is defined as the hour angle of the vernal equinox. When measured from the Greenwich meridian (i.e. the meridian of zero astronomic longitude) it is denoted as Greenwich Apparent Sidereal Time, GAST, and when measured from the local meridian it is called Local Apparent Sidereal Time, LAST. Apparent Sidereal Time, AST, corresponds to the hour angle of the apparent vernal equinox. The relationship between Local and Greenwich Sidereal Time is expressed as follows:

$$GAST = LAST + \Lambda_w \quad ,$$

$$GMST = LMST + \Lambda_w \quad ,$$

where Λ_w denotes the astronomic longitude of the local meridian west of the Greenwich meridian (not to be confused with ecliptic longitude).

Mean Sidereal Time, MST, is defined as the hour angle to the mean vernal equinox. The difference between Apparent and Mean Sidereal Time (i.e. between the apparent and mean vernal equinoxes) is known as the Equation of the Equinoxes, Eq.E. This difference is due to nutation (see Chapter 8) and is sometimes referred to as Nutation in Right Ascension. The Equation of the Equinoxes can be expressed in these terms as:

$$Eq.E = AST - MST = \Delta\Psi \cos \varepsilon \quad ,$$

where $\Delta\Psi$ is the nutation in ecliptic longitude. Greenwich Apparent Sidereal Time, GAST, and Greenwich Mean Sidereal Time, GMST, are the commonly tabulated quantities.

The variable, non-uniform rate of rotation of the Earth consequently renders this system impractical for measuring precise intervals of time.

3.3 SOLAR TIME

Solar time is classified as to whether it is based upon the motion of the apparent Sun or Newcomb's mean Sun.

Apparent Solar Time, AT, is defined by the apparent variable motion of the Sun as seen by an observer on the Earth. The epoch of AT is defined as 12h + local hour angle of the apparent Sun. Greenwich Apparent Solar Time, GAT, is referred to the Greenwich meridian but because of its non-uniformity, the hour angle is more commonly utilized to describe the location of the apparent Sun.

Mean Solar Time, MT, is the basis of all civil timekeeping. It is based upon the uniform, diurnal motion of the mean Sun whose right ascension, referred to the mean equinox of date, is given by Newcomb [1898a] as:

$$\alpha_m = 18.646066 + 2400.051262\ Tm + 0.000026\ Tm^2 \quad (hr)\ ,$$

where Tm is the interval of Julian centuries of 36525 mean solar days that have elapsed since the fundamental epoch of Universal Time; 1900 January 0.5d UT.

The definition of the epoch of MT is analogous to Apparent Solar Time (i.e. 12h + local hour angle of the mean Sun). Greenwich Mean Solar Time, GMT, or Universal Time, UT, is referred to the Greenwich meridian as 12h + Greenwich hour angle of the mean Sun.

The difference between Apparent and Mean Solar Times is called the Equation of Time, Eq.T, and is defined as:

$$Eq.T. = AT - MT = GHAa - GHAm = \alpha m - \alpha a\ ,$$

where 'a' refers to the apparent Sun and 'm' to the mean Sun.

Different classifications of Universal Time arise from considerations of the variable rate of rotation of the Earth and polar motion, i.e. the motion of the instantaneous spin axis with respect to the solid Earth. UT0 is observed Universal Time with no corrections applied. UT1 is corrected for polar motion and thus represents the Earth's true angular velocity. This is the time that is used for precise astronomical calcualations. UT2 is corrected for both polar motion and seasonal variations in the Earth's rotational speed. Although relatively uniform, UT2 is still subject to secular variations due to tidal forces and internal processes within the Earth [Thomson, 1978]. Since about 1962, UT2 has been superceded by Coordinated Universal Time, UTC, based on atomic clocks, as the most commonly broadcast time scale.

3.4 ATOMIC TIME

The desire for a more stable time system led to the introduction of atomic clocks in 1955. The duration of the atomic second was defined in 1967 by the International Committee for Weights and Measures as [Robbins,1976]:

> ...the duration of 9192631770 periods of radiation corresponding to the transition between the two hyper-fine levels of the fundamental state of the atom of Cessium 133.

Various systems of atomic time have been in use since 1955 but the internationally agreed upon system, known as International Atomic Time, TAI, was not introduced until 1972 [N.A.O., 1979a]. Coordinated Universal Time, UTC, is offset from TAI by an integral number of seconds as established also by international agreement. UTC is intentionally offset from TAI to keep it within $0^s.9$ of UT1. Today (1983), TAI-UTC=20^s.

The Bureau Internationale de Heure, BIH, is responsible for maintaining both TAI and UTC. Weekly publications by the BIH (e.g. B.I.H. [1983]) inform users of the current relationships between the time systems.

3.5 JULIAN DATES

It is often convenient to express an epoch in terms of its Julian date (JD), which is the interval of time in days and fractions of days since 4713 B.C., Jan. 1.5 days UT. This allows one to quickly determine the number of days between two epoch. Of importance for the calculations to be performed here are the 4 following Julian dates.

Julian Date	Epoch
2415020.0	1900, Jan. 0.5 days UT/ET
2433282.423	1950.0 (Bessilian Date)
2442413.478	1975.0 (Bessilian Date)

The Julian date may be computed for any epoch from the following algorithm from Meeus [1962]. Given the year (Y), month (M), day (D) and time (UT), the corresponding Julian date can be computed as follows:

If $M = 1$ or $M = 2$, $Y = Y - 1$ and

$M = M + 12$

$$A = INT(Y/100)$$

$$B = 2 - A + INT(A/4)$$

$$JD = INT(365.25\ Y) + INT(30.6001(M+1)) + D + UT/24 + 1720994.5 + B \quad ,$$

where INT denotes the integer operation (i.e. truncation).

3.6 RELATIONSHIPS BETWEEN TIME SYSTEMS

3.6.1 Ephemeris and Universal Time

There is no specific relationship between these systems. The difference is determined from astronomical observations on the planets (usually the moon) and is defined as:

$$\Delta T = ET - UT \quad .$$

This difference can be obtained from the publications of the BIH and United States Naval Observatory to one month in advance with a precision of $0\overset{s}{.}1$. More precise values must be determined from the publications of the B.I.H. one month in arrears.

3.6.2 Sidereal and Universal Time

The relationship between epochs may be determined from the following expression given by Newcomb [1898a] and adopted by the I.A.U.:

$$\begin{aligned} GMST &= UT + 12h + \alpha_m \\ &= UT + 6.646066 + 2400.051262\ Tm + 0.000026\ Tm^2 \ (hr), \end{aligned}$$

where Tm is the interval of Julian centuries elapsed since 1900 January 0.5d UT.

The conversion between intervals is determined from the ratio of the lengths of the sidereal and mean solar day and is expressed as [Mueller, 1969]:

$$ST\ /\ UT = 0.997269566414 - (0.586 \times 10^{-10})\ Tm \quad .$$

3.6.3 Atomic and Ephemeris Time

The difference between the duration of the atomic and ephemeris second is insignificant. Therefore, the difference between TAI and ET is a constant [N.A.O., 1979a]:

$$ET - TAI = 32\overset{s}{.}18 \quad .$$

The difference between ephemeris time and UTC can then be given as:

$$ET - UTC = 32\overset{s}{.}18 + (TAI - UTC) \quad .$$

Today (1983), ET-UTC=$54\overset{s}{.}18$, but as mentioned in 4.4, the difference (TAI-UTC) is periodically adjusted by an integral number of seconds.

3.6.4 Atomic and Universal Time

As stated above, the difference between UTC and UT1 is kept within $0\overset{s}{.}9$. The broadcasting stations encode the difference, DUT1 = UT1-UTC, within the time signal to an accuracy of $0\overset{s}{.}1$. DUT1 is published by the BIH one month in advance to a precision of $0\overset{s}{.}1$. More precise values are also published one month is arrears.

4. THE ORBITAL MOTION OF THE EARTH

The laws governing planetary motion in the solar system were discovered by Kepler. The first of these laws states that the orbit of a planet around the Sun is an ellipse, the position of the Sun being at a focus of the ellipse. It is known that the equation of an ellipse is [Smart, 1960]:

$$R = p / (1 + e \cos v)$$

where;

R = radius vector of orbit

$e = ((a^2-b^2)/a^2)^{1/2}$ = eccentricity of orbit

$p = a(1-e^2)$ = mean radius of orbit

a = semi-major axis of orbit

b = semi-minor axis of orbit

$v = \lambda - w$ = true anomaly

λ = ecliptic longitude of planet

w = ecliptic longitude of perihelion

These quantities are illustrated in Figure 5.

Kepler's second law states that the radius vector sweeps out equal areas in equal times and the third law asserts that the square of the orbital period is proportional to the cube of the length of the semi-major axis.

Theoretically, Kepler's second law permits the determination of the position of a planet in its orbit, given the semi-major axis, the eccentricity, the orbital period, P, and the time, t, at which the planet passed through perihelion [Smart, 1960].

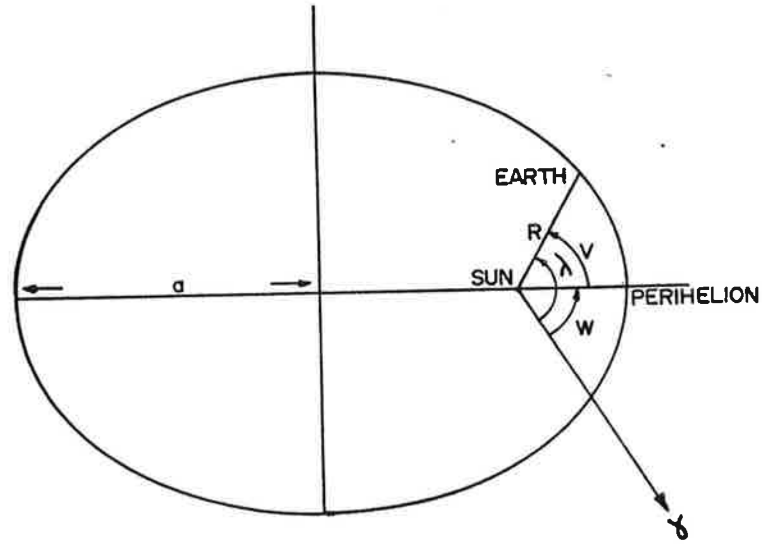

Figure 5: The orbital ellipse

The true anomaly, v, is often sought in terms of two other anomalies: the mean anomaly, Me, and the eccentric anomaly, E (see Figure 6). The mean anomaly is the angle measured from perihelion to Newcomb's mean Earth and the eccentric anomaly is measured from perihelion to the point of intersection with a line produced from the Earth perpendicular to the major axis and a circle of radius equal to the major semi-axis.

The eccentric anomaly is computed from the mean anomaly in an iterative manner using Kepler's Equation [Smart, 1960]:

$$Me = E - \sin E .$$

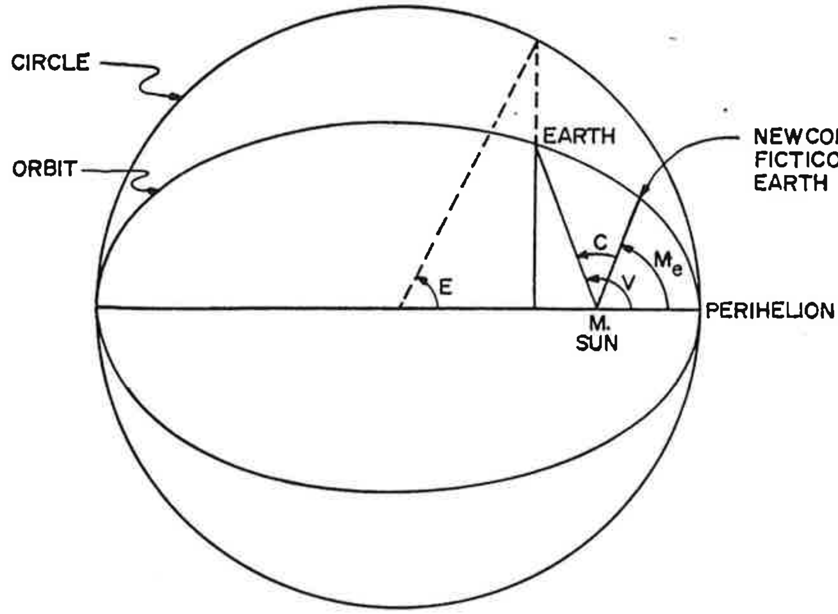

Figure 6: Anomalies

Subsequently, the true anomaly can be computed from the following relationship [Smart, 1960]:

$$\tan(v/2) = [(1+e)/(1-e)] \tan(E/2) .$$

If the Earth is considered to be at the origin of the ecliptic coordinate system, the Sun can be imagined to be orbiting about the Earth like a satellite. In this case, the point of closest approach of the Sun to the Earth is denoted as perigee and is in a direction opposite that of perihelion. Denoting w' as the ecliptic longitude of perigee and remembering that w is the ecliptic longitude of perihelion, it follows that

$$w' = w + 180^{\circ} \text{ (deg)} ,$$

and similarly

$$v' = v + 180^{o} \quad \text{(deg)} .$$

From Figure 7 it can be seen that the ecliptic longitude of the Sun, λ, may be expressed in terms of the mean orbital elements as follows:

$$\lambda = w' + v'$$

$$= L - Ms + v'$$

$$= L + C \quad ,$$

where $Ms = Me + 180^{o}$ is the apparent mean anomaly of the Sun. Newcomb [1898a] developed an expression for C, called the Equation of the Centre, in terms of the Sun's mean anomaly Ms. This is given as:

$$C = v - Me$$

$$= v' - Ms$$

$$= (1.9194603 - 0.0047889\ Te - 0.0000144\ Te^{2})\ \sin Ms +$$

$$+ (0.0200939 - 0.0001003\ Te)\ \sin 2Ms +$$

$$+ (0.0002928 - 0.0000003\ Te)\ \sin 3Ms +$$

$$+ 0.0000050\ \sin 4Ms \quad \text{(deg)} ,$$

where Te is the time in Julian ephemeris centuries elapsed since 1900 January 0.5d ET.

The mean orbital elements on which Newcomb has based his orbital theories are given as [Newcomb, 1898a]:

L = Sun's mean ecliptic longitude referred to the mean equinox of date

= 180^{o} + Earth's mean ecliptic longitude referred to the mean equinox of date

$$= 279.696678 + 36000.768925\ Te + 0.000303\ Te^{2} \quad \text{(deg)},$$

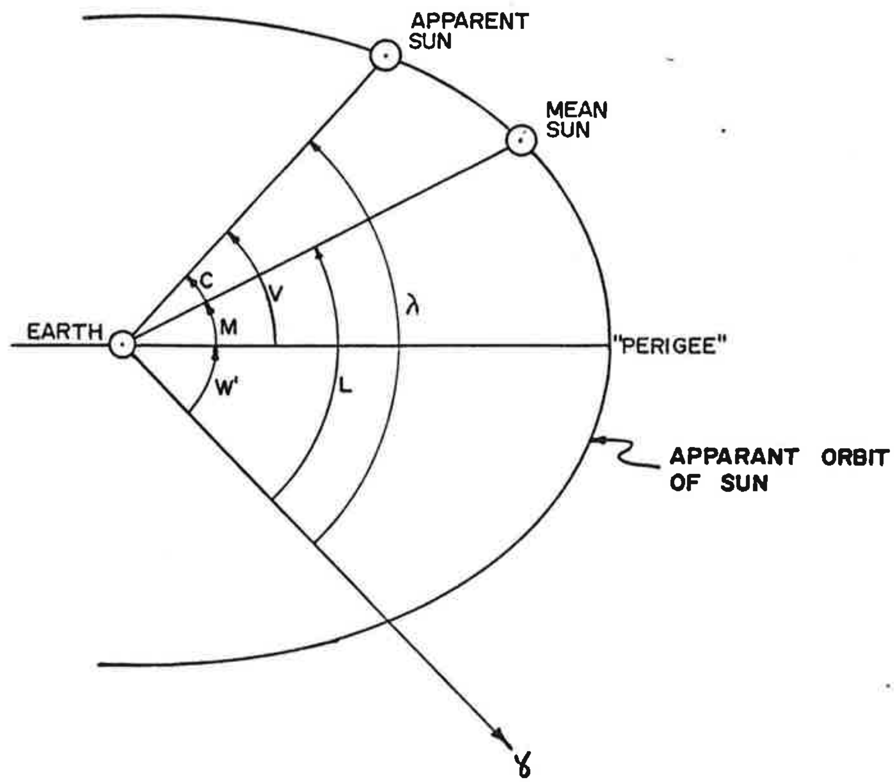

Figure 7: Newcomb's orbital elements

w = mean ecliptic longitude of perihelion referred to the mean equinox of date

$= 101.220833 + 1.719175\ Te + 0.000453\ Te^2 +$
$+ 0.000003\ Te^3$ (deg) ,

Ms = L - w' = mean anomaly of the Sun

$= 358.475833 + 35999.04975\ Te - 0.00015\ Te^2 -$
$- 0.000003\ Te^3$ (deg),

e = eccentricity

$= 0.01675104 - 0.00004180\ Te - 0.000000126\ Te^2$,

$\bar{\varepsilon}$ = mean obliquity of the ecliptic

$= 23.452294 - 0.013013\ Te - 0.000002\ Te^2$ (deg),

$\log\bar{R}$ = natural logarithm of the mean radius vector

(R in astronomical units)

$= 0.00003057 - 0.00000015\ Te\ +$

$+ (-0.00727412 + 0.00001814\ Te + 0.00000005\ Te^2)\cos Ms\ +$

$+ (-0.00009138 + 0.00000046\ Te)\cos 2Ms\ +$

$+ (-0.00000145 + 0.00000001\ Te)\cos 3Ms\ -$

$- 0.00000002\cos 4Ms$.

Again, Te denotes the interval of ephemeris centuries elapsed since 1900 Jan. 0.5d ET.

5. PERTURBATIONS

In the preceding chapter it was assumed that the orbital path of the Earth was determined by its mutual gravitational attraction with the Sun. But every other body in the solar system also affects, to some extent, the motion of the Earth and therefore the apparent motion of the Sun.

There are very extensive theories concerning these effects, known as perturbations, and it is beyond the scope of this report to discuss them here. Instead, the results of Newcomb's and LaVerrier's theories [Newcomb, 1891, 1898a], that have been exclusively used in preparing the fundamental ephemerides of the Sun, will be given.

Newcomb [1898a] has shown that the perturbations produced by a disturbing planet can be expressed as a sum of many periodic constituents, each reduced to the from

$$s \cos(K-jM-iMs) ,$$

where Ms is the mean anomaly of the Sun, M is the mean anomaly of the disturbing planet and s, K, j and i are constants for the specific periodic components given in Table 2. The constant 's' for the natural logarithm of the radius vector, logR, is expressed in units of the ninth decimal place.

Table 2 has omitted some long period terms in ecliptic longitude. Newcomb [1898a] has given the following expression for these effects where Te denotes the interval of ephemeris centuries elapsed since 1900 Jan. 0.5d ET:

The quantities upon which these expressions are based have been taken from Newcomb [1898a,b,c,d], the Nautical Almanac Offices [1961] and Meeus [1962] and are given as:

Ms = mean anomaly of the Sun (see Chapter 4),

$$Mmn = 296.104608 + 477198.849108\ Te + 0.009192\ Te^2 + 0.000014\ Te^3\ (deg)\ ,$$

M = mean anomaly of the disturbing planet

Mercury: $Mmc = 102.279381 + 149472.515289\ Te + 0.000507\ Te^2\ (deg)$

Venus: $Mv = 212.603222 + 58517.803875\ Te + 0.001286\ Te^2\ (deg)$

Mars: $Mm = 319.529022 + 19139.859219\ Te + 0.000181\ Te^2 + 0.000001\ Te^3\ (deg)$

Jupiter: $Mj = 225.32833 + 3034.96202\ Te - 0.000722\ Te^2\ (deg)$

Saturn: $Msn = 175.46622 + 1221.55147\ Te - 0.000502\ Te^2\ (deg)$

D = mean elongation of the Moon from the Sun

$$= 350.737486 + 445267.114217\ Te - 0.001436\ Te^2 + 0.000002\ Te^3\ (deg)\ ,$$

F = mean argument of ecliptic latitude of the Moon

= mean ecliptic longitude of the Moon - ecliptic longitude of the mean ascending node of the lunar orbit on the ecliptic (ε)

$$= 11.250889 + 483202.02515\ Te - 0.003211\ Te^2\ (deg)\ ,$$

u' = average distance of the Sun from the Moon's ascending node. (For practical purposes, the terms containing this argument may be neglected. Maximum errors resulting from this are 0."013 in ecliptic longitude, 0"036 in latitude and 0.000000003 in logR--i.e. approximately 7×10^{-9} AU),

$\log\overline{R}$ = logarithm of the mean radius vector (see Chapter 4),

From the results of Chapter 4 and the above, the ecliptic coordinates of the Sun may be expressed in the following manner:

λ = geometric ecliptic longitude of the Sun referred to the mean equinox of date

$= L + C + d\lambda$,

β = ecliptic latitude of the Sun referred to the mean equinox of date

$\doteq$ ecliptic latitude of the Sun referred to the true equinox of date (i.e. ecliptic latitude is insignificantly affected by nutation)

$= d\beta + \overline{\beta} \doteq d\beta$,

$\log R$ = natural logarithm of the radius vector (R in astronomical units)

$= \log\overline{R} + d(\log R)$,

where;

$\overline{\beta} = 0$ (deg) = mean ecliptic latitude of the Sun

$d\lambda$ = total perturbations in ecliptic longitude

$= d\lambda_{LP} + d\lambda mn + d\lambda mc + d\lambda v + d\lambda m + d\lambda j + d\lambda sn$

$d\beta$ = total perturbations in ecliptic latitude

$= d\beta mn + d\beta mc + d\beta v + d\beta m + d\beta j + d\beta sn$

d(logR) = total perturbations in logR

$= d(\log R)mn + d(\log R)mc + d(\log R)v + d(\log R)m +$

$+ d(\log R)j + d(\log R)sn$

TABLE 2

Perturbation constants (after Newcomb [1898a])

MERCURY - Longitude and Radius Vector Perturbations

		Longitude		Log Radius Vector	
j	i	s(")	K(deg)	s(10^{-9})	K(deg)
-1	1	0.013	243.000	28.000	335.000
-1	2	0.005	225.000	6.000	130.000
-1	3	0.015	357.000	18.000	267.000
-1	4	0.023	326.000	5.000	239.000

VENUS - Longitude and Radius Vector Pertubations

		LONGITUDE		LOG RADIUS VECTOR	
j	i	s(")	K(deg)	s(10^{-9})	K(deg)
-1	0	0.075	296.600	94.000	205.000
-1	1	4.838	299.102	2359.000	209.080
-1	2	0.074	207.900	69.000	348.500
-1	3	0.009	249.000	16.000	330.000
-2	0	0.003	162.000	4.000	90.000
-2	1	0.116	148.900	160.000	58.400
-2	2	5.526	148.313	6842.000	58.318
-2	3	2.497	315.943	869.000	226.700
-2	4	0.044	311.400	52.000	38.800
-3	2	0.013	176.000	21.000	90.000
-3	3	0.666	177.710	1045.000	87.570
-3	4	1.559	345.253	1497.000	255.250
-3	5	1.024	318.150	194.000	49.500
-3	6	0.017	315.000	19.000	43.000
-4	3	0.003	198.000	6.000	90.000
-4	4	0.210	206.200	376.000	116.280
-4	5	0.144	195.400	196.000	105.200
-4	6	0.152	343.800	94.000	254.800
-4	7	0.006	322.000	6.000	59.000
-5	5	0.084	235.600	163.000	145.400
-5	6	0.037	221.800	59.000	132.200
-5	7	0.123	195.300	141.000	105.400
-5	8	0.154	359.600	26.000	270.000
-6	6	0.038	264.100	80.000	174.300
-6	7	0.014	253.000	25.000	164.000
-6	8	0.010	230.000	14.000	135.000
-6	9	0.014	12.000	12.000	284.000
-7	7	0.020	294.000	42.000	203.500
-7	8	0.006	279.000	12.000	194.000
-7	9	0.003	288.000	4.000	166.000
-7	10	0.000	0.000	4.000	135.000
-8	8	0.011	322.000	24.000	234.000
-8	9	0.000	0.000	6.000	218.000
-8	12	0.042	259.200	44.000	169.700
-8	13	0.000	0.000	12.000	222.000
-8	14	0.032	48.800	33.000	138.700
-9	9	0.006	351.000	13.000	261.000
-9	10	0.000	0.000	4.000	256.000
-10	10	0.003	18.000	8.000	293.000

MARS - Longitude and Radius Vector Pertubations

j	i	LONGITUDE s(")	LONGITUDE K(deg)	LOG RADIUS VECTOR s(10^{-9})	LOG RADIUS VECTOR K(deg)
1	-2	0.006	218.000	8.000	130.000
1	-1	0.273	217.700	150.000	127.700
1	0	0.048	260.300	28.000	347.000
2	-3	0.041	346.000	52.000	255.400
2	-2	2.043	343.888	2057.000	253.828
2	-1	1.770	200.402	151.000	295.000
2	0	0.028	148.000	31.000	234.300
3	-4	0.004	284.000	6.000	180.000
3	-3	0.129	294.200	168.000	203.500
3	-2	0.425	338.880	215.000	249.000
3	-1	0.008	7.000	6.000	90.000
4	-4	0.034	71.000	49.000	339.700
4	-3	0.500	105.180	478.000	15.170
4	-2	0.585	334.060	105.000	65.900
4	-1	0.009	325.000	10.000	53.000
5	-5	0.007	172.000	12.000	90.000
5	-4	0.085	54.600	107.000	324.600
5	-3	0.204	100.800	89.000	11.000
5	-2	0.003	18.000	3.000	108.000
6	-6	0.000	0.000	5.000	217.000
6	-5	0.020	186.000	30.000	95.700
6	-4	0.154	227.400	139.000	137.300
6	-3	0.101	96.300	27.000	188.000
7	-6	0.006	301.000	10.000	209.000
7	-5	0.049	176.500	60.000	86.200
7	-4	0.106	222.700	38.000	132.900
8	-7	0.003	72.000	5.000	349.000
8	-6	0.010	307.000	15.000	217.000
8	-5	0.052	348.900	45.000	259.700
8	-4	0.021	215.200	8.000	310.000
9	-7	0.004	57.000	6.000	329.000
9	-6	0.028	298.000	34.000	208.100
9	-5	0.062	346.000	17.000	257.000
10	-7	0.005	68.000	8.000	337.000
10	-6	0.019	111.000	15.000	23.000
10	-5	0.005	338.000	0.000	0.000
11	-7	0.017	59.000	20.000	330.000
11	-6	0.044	105.900	9.000	21.000
12	-7	0.006	232.000	5.000	143.000
13	-8	0.013	184.000	15.000	94.000
13	-7	0.045	227.800	5.000	143.000
15	-9	0.021	309.000	22.000	220.000
15	-8	0.000	0.000	6.000	261.000
17	-10	0.004	243.000	4.000	153.000
17	-9	0.026	113.000	0.000	0.000

JUPITER - Longitude and Radius Vector Pertubations

j	i	LONGITUDE s(")	K(deg)	LOG RADIUS VECTOR s(10^{-9})	K(deg)
1	-3	0.003	198.000	5.000	112.000
1	-2	0.163	198.600	208.000	112.000
1	-1	7.208	179.532	7067.000	89.545
1	0	2.600	263.217	244.000	338.600
1	1	0.073	276.300	80.000	6.500
2	-3	0.069	80.800	103.000	350.500
2	-2	2.731	87.145	26.000	357.108
2	-1	1.610	109.493	459.000	19.467
2	0	0.073	252.600	8.000	263.000
3	-4	0.005	158.000	9.000	69.000
3	-3	0.164	170.500	281.000	81.200
3	-2	0.556	82.650	803.000	352.560
3	-1	0.210	98.500	174.000	8.600
4	-4	0.016	259.000	29.000	170.000
4	-3	0.044	168.200	74.000	79.900
4	-2	0.080	77.700	113.000	347.700
4	-1	0.023	93.000	17.000	3.000
5	-5	0.000	0.000	3.000	252.000
5	-4	0.005	259.000	10.000	169.000
5	-3	0.007	164.000	12.000	76.000
5	-2	0.009	71.000	14.000	343.000

SATURN - Longitude and Radius Vector Pertubations

j	i	LONGITUDE s(")	K(deg)	LOG RADIUS VECTOR s(10^{-9})	K(deg)
1	-2	0.011	105.000	15.000	11.000
1	-1	0.419	100.580	429.000	10.600
1	0	0.320	269.460	8.000	353.000
1	1	0.008	270.000	8.000	0.000
2	-3	0.000	0.000	3.000	198.000
2	-2	0.108	290.600	162.000	200.600
2	-1	0.112	293.600	112.000	203.100
2	0	0.017	277.000	0.000	0.000
3	-2	0.021	289.000	32.000	200.100
3	-1	0.017	291.000	17.000	201.000
4	-2	0.003	288.000	4.000	194.000

VENUS - Latitude Perturbations

j	i	LATITUDE s(")	K(deg)
-1	0	0.029	145.0
-1	1	0.005	323.0
-1	2	0.092	93.7
-1	3	0.007	262.0
-2	1	0.023	173.0
-2	2	0.012	149.0
-2	3	0.067	123.0
-2	4	0.014	111.0
-3	2	0.014	201.0
-3	3	0.008	187.0
-3	4	0.210	151.8
-3	5	0.007	153.0
-3	6	0.004	296.0
-4	3	0.006	232.0
-4	5	0.031	1.8
-4	6	0.012	180.0
-5	6	0.009	27.0
-5	7	0.019	18.0
-6	5	0.006	288.0
-6	7	0.004	57.0
-6	8	0.004	57.0
-8	12	0.010	61.0

MARS - Latitude Perturbations

j	i	LATITUDE s(")	K(deg)
2	-2	0.008	90.0
2	0	0.008	346.0
4	-3	0.007	188.0

JUPITER - Latitude Perturbations

j	i	LATITUDE s(")	K(deg)
1	-2	0.007	180.0
1	-1	0.017	273.0
1	0	0.016	180.0
1	1	0.023	268.0
2	-1	0.166	265.5
3	-2	0.006	171.0
3	-1	0.018	267.0

SATURN - Latitude Perturbations

j	i	LATITUDE s(")	K(deg)
1	-1	0.006	260.0
1	1	0.006	280.0

6. PROPER MOTION AND PRECESSION

The preceeding two chapters were concerned with the apparent average motion of the Sun. This chapter, on the other hand, primarily focuses on the mean motion of the stars outside our solar system. Since the concepts behind proper motion and precession have been exhaustively reported in many introductory textbooks on astronomy, only a brief outline of the ideas involved and the equations essential for applications to the problem at hand will be supplied (i.e. position updating).

6.1 PROPER MOTION

The postions of the stars with respect to each other have been observed to be variable. Each star appears to move in space as a result of its own actual motion and its apparent motion due to the motion of our solar system [Mueller,1969]. This total motion is called proper motion and is determined from astronomical observations.

The effects of proper motion on right ascension and declination are very small. For Polaris, the values of proper motions in right ascension and declination for the epoch 1950.0 are $18^{s}.1$ and $0''.43$ per tropical century respectively. The changes in the values of proper motion with respect to time are even smaller; $8^{s}.783$ per tropical century for right ascension and $-1''21$ per tropical century for declination. Note that the FK4 coordinates contain the e-terms of aberration. These must be removed when updating stars with large declinations (e.g. Polaris) due to the secant term in the elliptical aberration correction (see Chapter 8).

Given the right ascension (α_o) and declination (δ_o) of a star for epoch and mean equinox of t_o the coordinates (α'_o and δ'_o) for epoch t and mean equinox t_o may be determined by applying the star's proper motion. Denoting u to be proper motion in right ascension, u' to be proper motion in declination and du/dt and du'/dt to be the corresponding rates of change of the proper motions, these coordinates can be computed as follows:

$$\alpha'_o = \alpha_o + u(t-t_o) + 0.5\ du/dt\ (t-t_o)^2$$

$$\delta'_o = \delta_o + u'(t-t_o) + 0.5\ du'/dt\ (t-t_o)^2$$

where t and t_o are in tropical centuries.

6.2 PRECESSION

The attraction of celestial bodies on the Earth's equatorial bulge causes the rotational axis of the Earth to precess in a circular motion. The uniform, mean motion, with a period of about 25,800 years, is due to the Moon, Sun and planets and is known as general precession.

The effect of general precession on the coordinates of a celestial object is shown in Figure 8. At the initial epoch t_o, the right ascension, declination, vernal equinox, ecliptic, north celestial pole and north ecliptic pole is given as α_o, δ_o, Υ_o, ε_o, NCP_o and NEP_o respectively. For epoch t the subscripts are dropped.

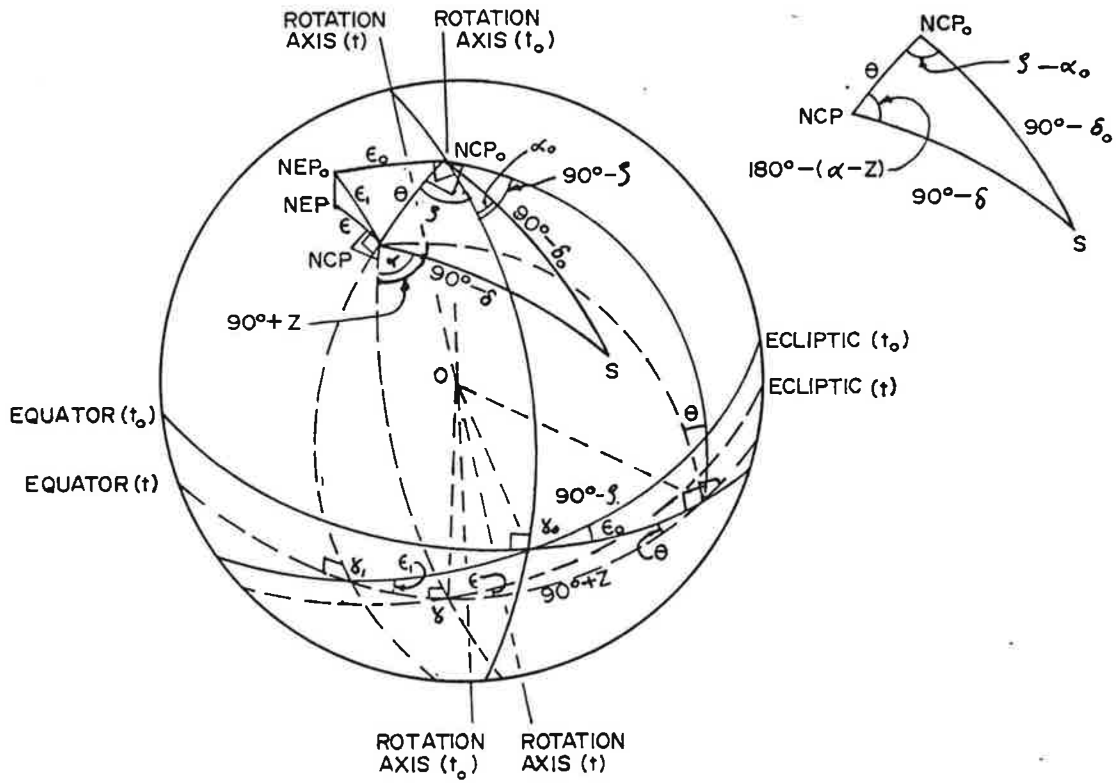

Figure 8: General Precession

The motion of the north celestial pole is described by the 3 angles ζ , z and θ which are called the precessional elements. Newcomb [1906] has derived expressions for these angles as functions of time based on both observations and theory. These are [N.A.O., 1961]:

$$\zeta = (2304.250 + 1.396t_o)t + 0.302t^2 + 0.018t^3 \text{ (arcsec)}$$

$$z = \zeta + 0.791t^2 + 0.001t^3 \text{ (arcsec)}$$

$$\theta = (2004.682 - 0.853t_o)t - 0.426t^2 - 0.042t^3 \text{ (arcsec)} ,$$

where the initial epoch t_0 is the number of tropical centuries (the difference between the tropical and Besselian interval is ignored here) elapsed since the Besselian epoch 1900.0 (JD 2415019.813) and the final epoch t is the number of tropical centuries elapsed since the initial epoch.

If the initial epoch is assumed to be 1950.0 (JD 2433282.423), the above expressions reduce to:

$$\zeta = 2304.948t + 0.302t^2 + 0.018t^3 \quad \text{(arcsec)}$$

$$z = \zeta + 0.791t^2 + 0.001t^3 \quad \text{(arcsec)}$$

$$\theta = 2004.255t - 0.426t^2 - 0.042t^3 \quad \text{(arcsec)} ,$$

where t is given by:

$$t = (JD-2433282.423)/36524.2199 \quad \text{(tropical centuries)} .$$

For the initial epoch 1975.0 (JD 2442413.478) the expressions become

$$\zeta = 2305.297t + 0.302t^2 + 0.018t^3 \quad \text{(arcsec)}$$

$$z = \zeta + 0.791t^2 + 0.001t^3 \quad \text{(arcsec)}$$

$$\theta = 2004.042t - 0.426t^2 - 0.042t^3 \quad \text{(arcsec)} ,$$

where

$$t = (JD - 2442413.478) / 36524.2199 \quad \text{(tropical centuries)}$$

From Figure 8 it is evident that the relationship between the two epochs may be developed from a series of rotations. To convert from the initial epoch to the final epoch it is required to first rotate the initial coordinate system about the z-axis by the angle $90^{\circ} - \zeta$. A subsequent rotation about the y-axis by θ is then needed and a final rotation of $-z$

about the z-axis will give the coordinate system in the final epoch. In matrix notation this is given as:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}_{RA'} = P(z,\theta,\zeta) \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} ,$$

where

$$\begin{aligned} P(z,\theta,\zeta) &= Rz(-z-90^\circ)Rx(\theta)Rz(90^\circ-\zeta) \\ &= Rz(-z)Ry(\theta)Rz(-\zeta) \\ &= \begin{bmatrix} P(1,1) & P(1,2) & P(1,3) \\ P(2,1) & P(2,2) & P(2,3) \\ P(3,1) & P(3,2) & P(3,3) \end{bmatrix} , \end{aligned}$$

and

$$P(1,1) = \cos z \cos\theta \cos\zeta - \sin z \sin\zeta$$

$$P(1,2) = -\cos z \cos\theta \sin\zeta - \sin z \cos\zeta$$

$$P(1,3) = -\cos z \sin\theta$$

$$P(2,1) = \sin z \cos\theta \cos\zeta + \cos z \sin\zeta$$

$$P(2,2) = -\sin z \cos\theta \sin\zeta + \cos z \cos\zeta$$

$$P(2,3) = -\sin z \sin\theta$$

$$P(3,1) = \sin\theta \cos\zeta$$

$$P(3,2) = -\sin\theta \sin\zeta$$

$$P(3,3) = \cos\theta \ .$$

Here, the R matrices are rotation matrices for a right-handed system of coordinates (cf. 2.4) and the argument in brackets is the angular value of the rotation.

After performing the necessary reductions, the following relationships are obtained:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}_{RA'} = \begin{bmatrix} \cos\delta' \cos\alpha' \\ \cos\delta' \sin\alpha' \\ \sin\delta' \end{bmatrix} = P(z,\theta,\zeta) \begin{bmatrix} \cos\delta_o \cos\alpha_o \\ \cos\delta_o \sin\alpha_o \\ \sin\delta_o \end{bmatrix} ,$$

and

$$\alpha' = \arctan(y'/x')$$

$$\delta' = \arcsin(z') \quad .$$

7. ASTRONOMIC NUTATION

In reality the precessional motion of the Earth's spin axis is not uniform. This irregular circular motion of the instantaneous spin axis of the Earth about the mean axis is called nutation and is due partly to the elliptical character of the Earth's orbit and to the inclination of the Moon's orbit with respect to the ecliptic [Mueller, 1969]. Astronomic nutation is commonly referred to as simply nutation and should not be confused with the free nutation or force-free precession of the Earth's spin axis about its principal moment of inertia axis [Mueller, 1969].

The principal term of astronomic nutation is produced by the inclination of the Moon's orbit with respect to the ecliptic. It thus depends on the ecliptic longitude of the Moon with a period of 18.6 years and has an amplitude of 9"210. This amplitude is often referred to as the constant of nutation. Other terms are due to the gravitational action of the Sun and the Moon on the non-spherical, rotating Earth. They depend on the mean ecliptic longitudes and mean anomalies of the Sun and Moon and their combinations with the ecliptic longitude of the Moon's node.

The resulting nutational motion of the pole of the instantaneous spin axis is resolved into two components; corrections to ecliptic longitude ($\Delta\Psi$) called nutation in ecliptic longitude and corrections to the obliquity ($\Delta\varepsilon$) called nutation in obliquity.

The theory and numerical series upon which nutation is presently based has been developed by Woolard [1953]. A newer theory also exists and is to be introduced into the fundamental emphemerides in 1984 (see Chapter 13). Here, Woolard's theory shall be used. The deviations of this with the modern approach will not be significant at the required level of precision. In this developement there are a total of 69 terms in $\Delta\Psi$ and 40 in $\Delta\varepsilon$ of which those with periods of less than 35 days are denoted as 'short-period' terms ($d\Psi$ and $d\varepsilon$); there are 46 short-period terms in $d\Psi$ and 24 in $d\varepsilon$

All long and short-period terms are listed in Table 3 which is reproduced from the Nautical Almanac Offices [1961]. The notation used in this table has been defined in previous chapters with the exception of Ω which is defined as:

Ω = longitude of the mean ascending node of the lunar orbit on the ecliptic

$$= 259.183275 - 1934.142008\ Te + 0.002078\ Te^2 + 0.000002\ Te^3 \quad (deg)\ ,$$

where Te is the interval of ephemeris centuries elapsed since 1900 Jan. 0.5d ET.

The procedure to follow when using the Table is to first compute the arguments to be used in the table. Next, for each row multiply each argument by its corresponding factor (i.e. columns 2 to 6) and sum them. This is to be used as the factor for the cosine function (for longitude) or the sine function (for obliquity). The corresponding arguments for the trigonometric functions are given in the last two columns. The sum of all sine terms for each period-row is the nutation in ecliptic longitude, $\Delta\Psi$, and the sum of all cosine terms is the nutation in obliquity, $\Delta\varepsilon$

As an example, for the period of 183.0 days the contributions to nutation in longitude ($\Delta\Psi'$) and obliquity ($\Delta\varepsilon'$). are:

$$\Delta\Psi' = (-1''2729-0''00013Te)\ \sin(2F-2D+2\Omega)$$

$$\Delta\varepsilon' = (0''5522-0''00029Te)\ \cos(2F-2D+2\Omega) \quad ,$$

where Te is the interval of ephemeris centuries elapsed since 1900 Jan. 0.5d ET.

Applying nutation to the ecliptic longitude of the Sun and obliquity of the ecliptic, both corrected for proper motion and precession, reduces both to the true equinox of date. Note that nutation does not affect the ecliptic latitude of the Sun.

For practical purposes (1" accuracy) only those terms whose coefficients are greater than $0''\!.1$ need to be considered.

The effect of nutation on the right ascension system may be derived in a manner similar to precession using rotation matrices. The resulting relationship is [Mueller, 1969]:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} = N(\varepsilon, \Delta\varepsilon, \Delta\Psi) \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}_{RA'} \quad ,$$

where

$$N(\varepsilon, \Delta\varepsilon, \Delta\Psi) = Rx(-\varepsilon-\Delta\varepsilon)Rz(-\Delta\Psi)Rx(\varepsilon)$$

$$= \begin{bmatrix} N(1,1) & N(1,2) & N(1,3) \\ N(2,1) & N(2,2) & N(2,3) \\ N(3,1) & N(3,2) & N(3,3) \end{bmatrix} ,$$

and

$$N(1,1) = \cos\Delta\Psi$$

$$N(1,2) = -\sin\Delta\Psi \cos\varepsilon$$

$$N(1,3) = -\sin\Delta\Psi \sin\varepsilon$$

$$N(2,1) = \cos(\varepsilon+\Delta\varepsilon) \sin\Delta\Psi$$

$$N(2,2) = \cos(\varepsilon+\Delta\varepsilon) \cos\Delta\Psi \cos\varepsilon + \sin(\varepsilon+\Delta\varepsilon) \sin\varepsilon$$

$$N(2,3) = \cos(\varepsilon+\Delta\varepsilon) \cos\Delta\Psi \sin\varepsilon - \sin(\varepsilon+\Delta\varepsilon) \cos\varepsilon$$

$$N(3,1) = \sin(\varepsilon+\Delta\varepsilon) \sin\Delta\Psi$$

$$N(3,2) = \sin(\varepsilon+\Delta\varepsilon) \cos\Delta\Psi \cos\varepsilon - \cos(\varepsilon+\Delta\varepsilon) \sin\varepsilon$$

$$N(3,3) = \sin(\varepsilon+\Delta\varepsilon) \cos\Delta\Psi \sin\varepsilon + \cos(\varepsilon+\Delta\varepsilon) \cos\varepsilon .$$

Here, RA' indicates the mean position of the RA system at the time of observation (i.e. corrected for precession).

The resulting expressions for the right ascension and declination referred to the true vernal equinox and ecliptic of date are then given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{RA} = \begin{bmatrix} \cos\delta \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \end{bmatrix} = N(\varepsilon, \Delta\varepsilon, \Delta\Psi) \begin{bmatrix} \cos\delta' \cos\alpha' \\ \cos\delta' \sin\alpha' \\ \sin\delta' \end{bmatrix} ,$$

and

$$\alpha = \arctan(y/x)$$

$$\delta = \arcsin(z) .$$

TABLE 3

Series for nutation (after N.A.O. [1961])

LONG-PERIOD TERMS

Period (days)	Argument Multiple of M	M	F	D	Ω	Longitude Coefficient of Sine argument		Obliquity Coefficient of Cosine argument	
						Unit=0"0001			
6798.0	0	0	0	0	1	-172327	-173.7 Te	92100	9.1 Te
3399.0	0	0	0	0	2	2088	0.2 Te	-904	0.4 Te
1305.0	-2	0	2	0	1	45	0.0	-24	0.0
1095.0	2	0	-2	0	0	10	0.0	0	0.0
6786.0	0	-2	2	-2	1	-4	0.0	2	0.0
1616.0	-2	0	2	0	2	-3	0.0	2	0.0
3233.0	1	-1	0	-1	0	-2	0.0	0	0.0
183.0	0	0	2	-2	2	-12729	-1.3 Te	5522	-2.9 Te
365.0	0	1	0	0	0	1261	-3.1 Te	0	0.0
122.0	0	1	2	-2	2	-497	1.2 Te	216	-0.6 Te
365.0	0	-1	2	-2	2	214	-0.5 Te	-93	0.3 Te
178.0	0	0	2	-2	1	124	0.1 Te	-66	0.0
206.0	2	0	0	-2	0	45	0.0	0	0.0
173.0	0	0	2	-2	0	-21	0.0	0	0.0
183.0	0	2	0	0	0	16	-0.1 Te	0	0.0
386.0	0	1	0	0	1	-15	0.0	8	0.0
91.0	0	2	2	-2	2	-15	0.1 Te	7	0.0
347.0	0	-1	0	0	1	-10	0.0	5	0.0
200.0	-2	0	0	2	1	-5	0.0	3	0.0
347.0	0	-1	2	-2	1	-5	0.0	3	0.0
212.0	2	0	0	-2	1	4	0.0	-2	0.0
120.0	0	1	2	-2	1	3	0.0	-2	0.0
412.0	1	0	0	-1	0	-3	0.0	0	0.0

SHORT-PERIOD TERMS

Period (days)	Argument Multiple of M	M	F	D	Ω	Longitude Coefficient of Sine argument		Obliquity Coefficient of Cosine argument	
						Unit=0"0001			
13.7	0	0	2	0	2	-2037	-0.2 Te	884	-0.5 Te
27.6	1	0	0	0	0	675	0.1 Te	0	0.0
13.6	0	0	2	0	1	-342	-0.4 Te	183	0.0
9.1	1	0	2	0	2	-261	0.0	113	-0.1 Te
31.8	1	0	0	-2	0	-149	0.0	0	0.0
27.1	-1	0	2	0	2	114	0.0	-50	0.0
14.8	0	0	0	2	0	60	0.0	0	0.0
27.7	1	0	0	0	1	58	0.0	-31	0.0
27.4	-1	0	0	0	1	-57	0.0	30	0.0
9.6	-1	0	2	2	2	-52	0.0	22	0.0
9.1	1	0	2	0	1	-44	0.0	23	0.0
7.1	0	0	2	2	2	-32	0.0	14	0.0
13.8	2	0	0	0	0	28	0.0	0	0.0
23.9	1	0	2	-2	2	26	0.0	-11	0.0
6.9	2	0	2	0	2	-26	0.0	11	0.0
13.6	0	0	2	0	0	25	0.0	0	0.0
27.0	-1	0	2	0	1	19	0.0	-10	0.0
32.0	-1	0	0	2	1	14	0.0	-7	0.0
31.7	1	0	0	-2	1	-13	0.0	7	0.0
9.5	-1	0	2	2	1	-9	0.0	5	0.0
34.8	1	1	0	-2	0	-7	0.0	0	0.0
13.2	0	1	2	0	2	7	0.0	-3	0.0
9.6	1	0	0	2	0	6	0.0	0	0.0
14.8	0	0	0	2	1	-6	0.0	3	0.0
14.2	0	-1	2	0	2	-6	0.0	3	0.0
5.6	1	0	2	2	2	-6	0.0	3	0.0
12.8	2	0	2	-2	2	6	0.0	-2	0.0
14.7	0	0	0	-2	1	-5	0.0	3	0.0
7.1	0	0	2	2	1	-5	0.0	3	0.0
23.9	1	0	2	-2	1	5	0.0	-3	0.0
29.5	0	0	0	1	0	-4	0.0	0	0.0
15.4	0	1	0	-2	0	-4	0.0	0	0.0
29.8	1	-1	0	0	0	4	0.0	0	0.0
26.9	1	0	-2	0	0	4	0.0	0	0.0
6.9	2	0	2	0	1	-4	0.0	2	0.0
9.1	1	0	2	0	0	3	0.0	0	0.0
25.6	1	1	0	0	0	-3	0.0	0	0.0
9.4	1	-1	2	0	2	-3	0.0	0	0.0
13.7	-2	0	0	0	1	-2	0.0	0	0.0
32.6	-1	0	2	-2	1	-2	0.0	0	0.0
13.8	2	0	0	0	1	2	0.0	0	0.0
9.8	-1	-1	2	2	2	-2	0.0	0	0.0
7.2	0	-1	2	2	2	-2	0.0	0	0.0
27.8	1	0	0	0	2	-2	0.0	0	0.0
8.9	1	1	2	0	2	2	0.0	0	0.0
5.5	3	0	2	0	2	-2	0.0	0	0.0

8. ABERRATION

Aberration is the angular displacement of the apparent position of a celestial object due to the finite velocity of light and the relative motion of the object and observer. The part that is due to the motion of the observer is called stellar aberration and that due to the motion of the object is referred to as the correction for light time. The combined effect of both stellar aberration and the correction for light time is known as planetary aberration.

Stellar aberration consists of the following three componets:

1. Diurnal Aberration - due to the rotation of the Earth.
2. Annual Aberration - due to the orbital motion of the Earth around the centre of mass of the solar system.
3. Secular Aberration - due to the motion of the solar system around the centre of the galaxy. This effect is included within proper motion.

When dealing with problems concerning the Sun and planets, only diurnal and annual aberration are considered since these completely describe the total relative motions.

8.1 ANNUAL ABERRATION

Annual aberration is computed from the actual motion of the Earth, referred to an inertial frame of reference and the centre of mass of the solar system, in accordance with the recommendations of the International Astronomical Union [1950, 1954].

Considering the Earth's orbit to be circular, it can be seen from Figure 9 that the instantaneous velocity vector of the Earth is in the direction $\lambda_s - 90^\circ$ (λ_s is the ecliptic longitude of the Sun). According to the general law of aberration, the displacement of an object with ecliptic longitude λ is in the same direction of the velocity of the observer. The angular displacement in ecliptic longitude at unit distance (1 AU) is given by Smart [1960] as:

$$\Delta\lambda = -k \sec\beta \cos(\lambda_s - \lambda) \quad \text{(arcsec)}$$

where k is the constant of aberration at unit distance R'=1AU, whose value is 20"496 [N.A.O., 1979a]. For the Sun $\beta=0$, $\lambda=\lambda_s$ and the effect on ecliptic longitude at distance R from the Sun is:

$$\Delta\lambda = -k(R'/R) \quad \text{(arcsec)} .$$

For stars it is generally more convenient to give the corrections to right ascension $\Delta\alpha$ and declination $\Delta\delta$. The fundamental aberration equation is given by Mueller [1969] as

$$\Delta\theta = \theta - \theta' = k \sin\theta', \quad \text{for small } \Delta\theta,$$

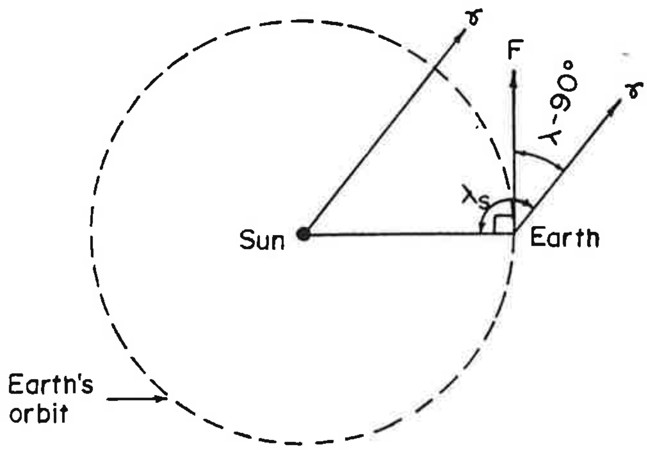

Figure 9: Annual aberration--circular orbit

where θ is the spatial angle between the true position of the star and the direction of motion of the Earth and θ' is the spatial angle between the apparent position of the star and the direction of motion of the Earth. The resulting corrections to right ascension $\Delta\alpha$ and declination $\Delta\delta$ can be shown to be

$$\Delta\alpha = -k \sec\delta' (\cos\lambda_s \cos\epsilon \cos\alpha' + \sin\lambda_s \sin\alpha')$$

$$\Delta\delta = -k[\cos\lambda_s \cos\epsilon(\tan\epsilon \cos\delta' - \sin\delta' \sin\alpha') + $$
$$+ \cos\alpha' \sin\delta' \sin\lambda_s] \quad ,$$

where α' and δ' are the apparent right ascension and declination of the star. The solution to this may be performed in an iterative manner using the true right ascension and declination as a first approximation to their apparent couterparts. Normally, no iterations are needed. However, for stars with large declinations (e.g. Polaris) the error in the $\sec\delta'$ term of $\Delta\alpha$ becomes significant and at least one iteration should be made (one iteration is generally sufficient).

When the Earth's elliptical orbit is considered another correction is applied, sometimes referred to as the 'e-terms of aberration'. The velocity of the Earth is resolved into a component perpendicular to the radius vector, F, and a component parallel to the minor axis, f, as illustrated in Figure 10. Smart [1960] has then shown that both F and f are constant along the orbit and that $f=eF$, where e is the eccentricity of the orbit (see Chapter 4). The direction of f is defined by an ecliptic longitude of $90^\circ + w$, w being the ecliptic longitude of perihelion of the Earth's orbit (not to be confused with w', the ecliptic longitude of perigee - see Chapter 4).

It may be shown, by replacing $w'-90^\circ$ with $\lambda_s - 90^\circ$ in the previous equations, that the effect in the ecliptic longitude of the Sun at unit distance R' is [Smart, 1960]:

$$\Delta\lambda = -ek \sec\beta \cos(w' - \lambda_s)$$

$$= -ek \cos(w' - \lambda_s) \quad \text{(arcsec)} ,$$

where $\beta \doteq 0$. At a distance R, therefore, the resulting correction is:

$$\Delta\lambda = -(R'/R)ek \cos(w' - \lambda_s) \quad \text{(arcsec)} ,$$

where R and R' are in the same units of length.

The effects in right ascension and declination are obtained by simply replacing λ_s with w' in the equations for the circular component. The maximum value of this correction is of the order of 0".34 (i.e. the value of ek) for stars where $|\delta| < 80^{\circ}$ and is therefore often ignored for accuracies of the order of 1". However, for Polaris, δ is very close to 90° and thus the relatively large value of the $\sec\delta$ term produces a significant correction.

This effect is usually ignored in the precise reduction of star coordinates obtained from the FK4 Star Catalog [Fricke and Kopff, 1963] as the e-terms are included in the tabulated values. The variation in these terms since the date of tabulation is significant only for stars with large declinations or when high accuracy is mandatory.

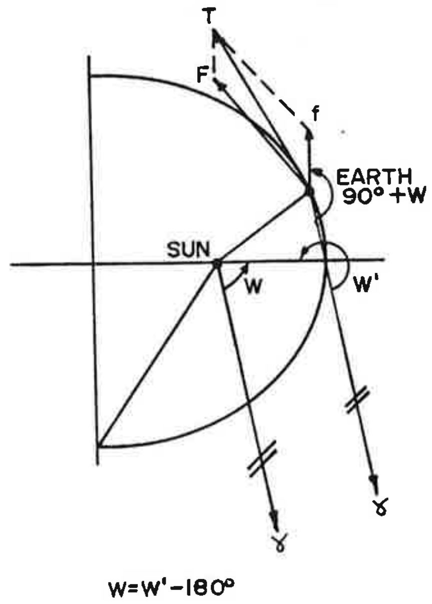

Figure 10: General aberration

8.2 DIURNAL ABERRATION

When dealing with stars that are close to the north celestial pole, the effect of diurnal aberration becomes significant for astronomic latitudes less than approximately 70° and must be taken into account.

The corrections to right ascension and declination have been derived by Mueller [1969] and are as follows:

$$\Delta\alpha = 0.021\rho \cos\Phi \cos h' \sec\delta' / \rho' \quad \text{(sec)} ,$$

$$\Delta\delta = 0.32\rho \cos\Phi \sin h' \sin\delta' / \rho' \quad \text{(arcsec)} ,$$

where ρ is the geocentric radius of the observer, ρ' is the geocentric radius of the Earth, Φ is the astronomic latitude of the observer, h' is the dispaced hour angle of the star and δ' is the displaced declination. The corrections should be added to the true directions to obtain the displaced positions. Again, the solution of these equations should be peformed iteratively for greatest accuracy, using the hour angle and declination unaffected by diurnal aberration as a first approximation. Only one iteration is usually required.

The significance of the corrections for circumpolar stars may be realized by substituting typical values for the parameters. Letting

$$\delta' = 89^\circ$$

$$\phi = 45^\circ$$

$$h = 46^\circ$$

$$\rho = \rho' \ ,$$

we find that $\Delta\alpha = 0^s.6$ which is significant for precise azimuth determinations.

9. PARALLAX

Parallax is due to the displacement of the observer from the origin of the coordinate system. It results in an apparent displacement of the observed position of a celestial object equal to the parallatic angle, defined as the angle subtended at the celestial object between the observer and the origin of the coordinate system.

There are two types of parallax due to the different motions of the Earth. Geocentric parallax is the angle subtended at the object between the direction of the observer and the centre of the Earth. Annual or stellar parallax is the angle at the object between the direction of the centre of mass of the Earth and the centre of mass of the solar system (i.e. the centre of the ecliptic coordinate system). For stellar observations both parallactic effects are very small [Mueller, 1969] and may be neglected here. Since the Sun can be considered to be at the centre of mass of the solar system, annual parallax is practically non-existent for solar observations and will also be neglected in these calculations. Therefore, only geocentric parallax as it relates to observations on the Sun will be discussed. Furthermore, geocentric parallax affects only the observed zenith distance significantly and is applied only for zenith distance azimuth observations.

Geocentric parallax is illustrated in Figure 11 where ρ is the distance of the observer from the centre of the

Earth, z' is the observed zenith distance, z is the geocentric zenith distance, a is the equatorial radius of the Earth (6378137 m), R is the distance of the object (the Sun) from the centre of the Earth (note - a and R must be in the same units of length) and π is the geocentric parallax. It can then be seen from Figure 11 that the following relationships exist between the observed and geocentric zenith distances:

$$z' = z + \pi$$

and

$$\sin\pi = \rho \sin z' / R$$

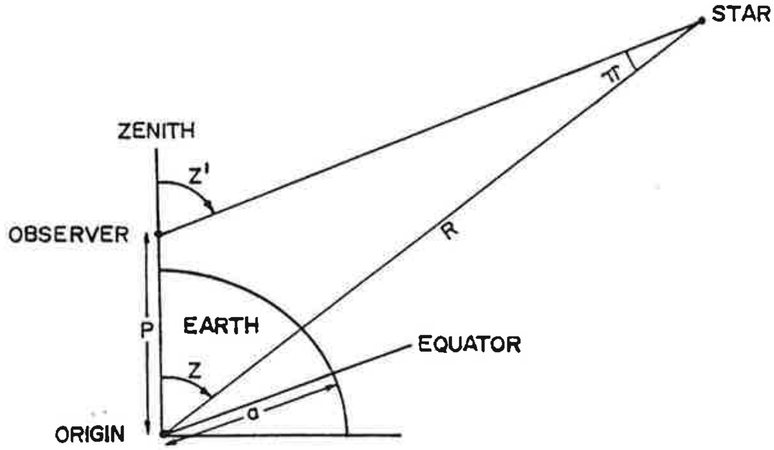

Figure 11: Geocentric Parallax

When the object is on the horizon (i.e. z'=90°) the parallax is denoted as horizontal parallax. If the observer is also at a distance $\rho = a$ the resulting parallax is known as equatorial horizontal parallax, π_o, and is expressed as:

$$\sin\pi_o = a / R$$

where a and R are in the same units of length.

At a constant unit distance R'=1Au the equatorial parallax is 8"794 and is called the constant of parallax, Π. The equatorial horizontal parallax at a distance R may then be determined from the constant of parallax by equating the equatorial distance a to give:

$$\sin\pi_o = (R'/R) \sin \Pi \ .$$

Similarly, the constant of parallax may be used to compute the geocentric parallax as follows:

$$\begin{aligned} \sin\pi &= (\rho/R) \sin z' \\ &\doteq (a/R) \sin z' \\ &= \sin \pi_o \sin z' \\ &= (R'/R) \sin\Pi \sin z' \ , \end{aligned}$$

where the error involved in approximating ρ with a is negligable on the Earth's surface (i.e. less than 0"1).

Finally, the geocentric zenith distance may be determined from:

$$\begin{aligned} z &= z' - \pi \\ &\doteq z' - \arcsin[(R'/R) \sin\Pi \sin z'] \ . \end{aligned}$$

10. SEMI-DIAMETER

The semi-diameter of the Sun is the apparent radius as seen from the Earth. It is obtained by dividing the adopted value of the semi-diameter at unit distance R'=1AU, with an allowance for irradiation, by the true radius vector. The adopted value at unit distance, called the constant of semi-diameter, is given as 16'01".18 [N.A.O., 1979]. The semi-diameter at a distance R can then be determined by:

$$S.D. = 0.266994\ (R'/R)\quad (deg)\ .$$

If a horizontal angle is measured only to the edge of the Sun, the so-called semi-diameter correction to the horizontal angle is given as:

$$\Delta HA = S.D.\ /\ \cos a\quad (deg)\ ,$$

where a is the altitude of the Sun. The sign of the correction depends upon which edge is observed; if observing the trailing edge the correction is added to the observed clockwise horizontal angle.

11. PRECISION CONSIDERATIONS

The Sun's astronomical data are presently tabulated to two levels of precision by the various ephemerides (see Table 4). The Astronomical Almanac publishes right ascension to a precision of $0^s.01$, Greenwich sidereal time to $0^s.001$, declination to $0''.1$ and semi-diameter to $0''.01$. The equation of time (E=12hr+Eq.T in The Star Almanac) is tabulated to a precision of $0^s.1$ in the K&E Ephemeris and Star Almanac. Both tabulate declination and semi-diameter to $0'.1$.

The apparent right ascension and declination of Polaris has been given much more precisely in the various star catalogues. Such precision, though, is not generally required for users such as land surveyors. Consequently, we have limited the precision to 1" in both right ascension and declination.

TABLE 4

Precision of ephemerides

Source	Quantity	Precision
The Astronomical Almanac	GAST	$0^s.001$
	RA	0.01
	Dec	0.1
	S.D.	0.01
The Star Almanac for Land Surveyors	E(GHA Sun)	$0^s.1$
	Dec	0.1
	S.D.	0.1
K & E Ephemeris	Eq.T.	$0^s.1$
	Dec	0.1
	S.D.	0.1

As previously stated, the expressions given in this report will produce a precision comparable to the published ephemerides. If such accuracy is not required, the expressions may be truncated to give the desired precision. Care must be exercised, however, when neglecting some of the smaller periodic perturbations since the large number of seemingly insignificant terms may accumulate into a relatively large correction.

12. SUMMARY

The results of the foregoing are summarized below as a series of steps to be executed when computing the azimuth of Polaris and the Sun. For each step references are made to the appropriate sections.

12.1 SUN

1. Compute the mean ecliptic longitude, L, mean log of the radius vector, $\log\overline{R}$, mean obliquity of the ecliptic, $\overline{\varepsilon}$, and the equation of the centre, C - Chapter 4.
2. Compute the total perturbations in ecliptic longitude, $d\lambda$, ecliptic latitude, $d\beta$, and logR, d(logR), due to the planets and Moon - Chapter 5.
3. Compute the nutation in ecliptic longitude, $\Delta\Psi$, and obliquity, $\Delta\varepsilon$ - Chapter 7.
4. Compute the annual abberation correction to ecliptic longitude, $\Delta\lambda$ - Chapter 8.
5. Determine the apparent geocentric ecliptic coordinates of the Sun as follows:

λ = apparent geocentric ecliptic longitude

$= L + C + d\lambda + \Delta\Psi + \Delta\lambda$

β = apparent geocentric ecliptic latitude

$\doteq d\beta$

logR = log of the true radius vector

$= \log\overline{R} + d(\log R)$

ε = true obliquity of the ecliptic

$= \bar{\varepsilon} + \Delta\varepsilon$

6. Compute the apparent geocentric right ascension and declination from the ecliptic coordinates - Chapter 2.
7. Compute the geocentric parallax and apply the correction to the observed zenith distance - Chapter 9.
8. For observations on the edge of the Sun compute the semi-diameter and apply the correction to the observed horizontal angle - Chapter 10.
9. Compute Greenwich apparent sidereal time and local hour angle - Chapters 2 and 3.

   $h = GAST - \alpha$
10. Compute azimuth of the Sun - Chapter 2.

12.2 POLARIS

1. Compute e-terms of aberration for the catalogued epoch and remove from the catalogued positions - Chapter 8.
2. Compute and apply proper motion to the 1950.0 right ascension and declination - Chapter 6.
3. Compute and apply precession from 1950.0 to date of observation - Chapter 6.
4. Compute and apply nutation in right ascension and declination - Chapter 7.

5. Compute ecliptic longitude of the Sun and obliquity of the ecliptic for the following step - Chapters 4, 5 and 6.
6. Compute and apply annual and diurnal aberration using ecliptic longitude of the Sun and obliquity - Chapter 8.
7. Compute Greenwich apparent sidereal time and hour angle of Polaris - Chapters 2 and 3.

   $$h = GAST - \alpha$$

8. Compute azimuth of Polaris - Chapter 2.

13. I.A.U. IMPROVEMENTS TO THE ASTRONOMICAL CONSTANTS

The system of astronomical constants used in this report has been adopted by the General Assembly of the I.A.U. at Hamburg, September, 1964 [International Astronomical Union, 1966] and is the system currently in use. A list of the pertinent constants is given in Table 5. The complete list may be found in The Supplement to the American Ephemeris, 1968, pp.4s-7s. It should be noted that the adopted planetary mass ratios have not been incorporated into Newcomb's perturbation theories.

In 1976 the General Assembly of the I.A.U. adopted a set of recommendations calling for a re-definition of the astronomical constants. The changes are planned to be introduced in 1984. The following is a brief summary of the adopted recommendations:

1. A new fundamental epoch designated as J2000.0 (2000 January 1.5d UT or JD2451545.0) and the Julian century will be regarded as the unit of time in the equations of motion.
2. A new system of astronomical constants including changes to the constants of precession, nutation, aberration and parallax based on the fundamental epoch (see Table 6).
3. A new fundamental reference frame defined by the FK5 incorporating an equinox adjustment, which is also to be used to amend Greenwich mean sidereal time at zero

TABLE 5

I.A.U. 1964 system of astronomical constants (after N.A.O. [1961])

Defining Constant	
Gaussian gravitational constant	k=0.017202098950000
Primary Constants	
Astronomical unit	1.49600×10^{11} m
Velocity of light	2.997925×10^{8} m/s
Equatorial radius of Earth	6378160 m
Dynamical form-factor for Earth	0.0010827
Geocentric gravitational constant	3.98603×10^{14} m
Earth/Moon mass ratio	81.30
General precession in longitude per tropical century (1900)	5025".64
Constant of nutation (1900)	9".210
Obliquity of ecliptic (1900)	23°27' 08".258
Derived Constants	
Solar parallax	8".794
Constant of aberration	20".496
Light-time for unit distance	499.012 s
Flattening factor for Earth	1/298.25
Heliocentric gravitational constant	1.32718×10^{20} m^3/s^2
Sun/Earth mass ratio	332958
Sun/(Earth+Moon) mass ratio	328912

TABLE 6

I.A.U. 1976 system of astronomical constants (after Stein [1982])

Defining Constant	
Gaussian gravitational constant	k=0.01720209895000
Primary Constants	
Astronomical unit	$1.49597870 \times 10^{11}$ m
Velocity of light	2.99792458×10^{8} m/s
Equatorial radius of Earth	6378140 m
Dynamical form-factor for Earth	0.00108263
Geocentric gravitational constant	3.986005×10^{14} m
Earth/Moon mass ratio	81.3007
General precession in longitude per tropical century (2000)	5029″.0966
Constant of nutation (2000)	9″.2109
Obliquity of ecliptic (2000)	23° 26' 21″.448
Derived Constants	
Solar parallax	8″.794148
Constant of aberration	20″.49552
Light-time for unit distance	499.004782 s
Flattening factor for Earth	1/298.257
Heliocentric gravitational constant	$1.32712438 \times 10^{20}$ m^3/s^2
Sun/Earth mass ratio	332946.0
Sun/(Earth+Moon) mass ratio	328900.5

hr UT in order to avoid a discontinuity in UT. The expression for the correction to the FK4 equinox is [Fricke, 1980]:

$$E = 0.035 + 0.085(T-19.50) \quad (sec) \;,$$

where T is in Julian centuries. The corresponding expression for GMST at 0 hour UT is:

$$GMST(0hrUT) = 6.6973758 + 2400.0513372\ Tu + 0.0000258\ Tu^2 \quad (hr) \;,$$

where Tu is the interval of Julian centuries elapsed since 2000 January 1.5d UT1 (negative for years prior to 2000).

4. The 1980 I.A.U. Theory of Nutation based on Wahr's [1981] theories shall supercede Woolard's theories.

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Appendix A

PROGRAM SPADE - SOLAR AND POLARIS AZIMUTH DETERMINATION

This Appendix provides a description of the program SPADE which computes, among other things, the azimuth to a reference object from astronomical observations on either the Sun or Polaris. A listing of the program is given in Appendix C. The azimuth of the celestial object is determined from 2.4.2. Both the hour angle and zenith distance solutions have been incorporated for solar observations but only the hour angle solution has been included for observations on Polaris.

The program has been developed in Watfiv (WATerloo Fortran version IV) and is based upon 'stand-alone' subroutines in order to make specific individual modifications as simple as possible. No difficulties should be encountered running the program with standard versions of the Fortran compiler.

The second version of this program is given in this appendix. Changes were made to the original version to improve accuracy and portability. The main program has been divided into three basic parts. The first calculates various astronomical quantities for use in updating both the Sun and Polaris. The second part computes the azimuth of the Sun (and subsequently of the reference object) by deriving the Sun's right ascension and declination and various other data from the expressions given in the body of this report. The third part computes the azimuth of Polaris by updating its FK-4 coordintates in the traditional manner also outlined in this report.

A description of the notation used is supplied in the program and subroutine comment statements. In addition, complete input instructions are also provided within the program. Briefly, the input deck requires each individual observation to begin on a new record (i.e. on a new line or card). Furthermore, format-free input has been utilized requiring only a blank space to separate each data value.

The main program requires angular data to be input as degrees, minutes and seconds or, in the case of time arguments and right ascension, hours, minutes and seconds. However, all subroutines require angles to be given in decimal degrees or hours. Double precision variables of sixteen sigificant digits (i.e. variables that occupy eight bytes of memory instead of four bytes for 'real' variables) are used throughout.

To facilitate the simplification or modification of the program to either solar or polaris observations, a list of subroutines required for either observation is given below. A description of these is given in the program listing. Here, the letter 'P' indicates the subroutine is required for a polaris observation and "S" for a solar observation.

Polaris & Solar Observation Subroutines:

- AZHA, SOLDAT, DEG, DMS, GST, JDATE

Solar Observation Subroutines:

- AZZD, RADS, SDC

Polaris Observation Subroutines:

- AAB, DAB, NUT, PM, PREC

Appendix B

PROGRAM TESTING

In order to test the program, various examples were input and the results compared to manual computations. In the beginning, the coordinate calculations were compared to the published ephemerides and star catalogues. In all cases perfect agreement was found at the desired 1" level for a wide range of epochs (1960-1982). Subsequent tests involved checking the azimuth subroutines (i.e. AZZD and AZHA). Both operated correctly. The final tests concerned the proper operation of the complete SPADE program. Again, the program functioned correctly. A number of different observation types were input and checked against manual calculations to ensure perfect agreement.

Two examples of the output are given in the following Table.

TABLE 7

Sample SPADE Output

Example 1:

```
SPADE - SOLAR AND POLARIS AZIMUTH DETERMINATION


SOLAR OBSERVATION - HOUR ANGLE SOLUTION
WITH SEMI-DIAMETER CORRECTION

INPUT
    LATITUDE (D-M-S) =          43-40-10.0
    LONGITUDE (D-M-S) =         79-30- 0.0
    DATE (Y/M/S) =              1972/11/20
    UNIVERSAL TIME (H-M-S) = 20-10-20.0
    HORIZ ANGLE (D-M-S) =      210-10-20.0
OUTPUT
    ZENITH DIST (D-M-S) =       76-32-55.9
    GAST (H-M-S) =               0-10- 4.1
    RA (H-M-S) =                15-45-31.4
    DEC (D-M-S) =              -19-51-17.8
    AZ OF STAR (D-M-S) =       224-40-29.1
    AZ OF RO (D-M-S) =          14-13-28.6
```

Example 2:

```
SPADE - SOLAR AND POLARIS AZIMUTH DETERMINATION


POLARIS OBSERVATION

INPUT
    LATITUDE (D-M-S) =          43-40-10.0
    LONGITUDE (D-M-S) =         79-30- 0.0
    DATE (Y/M/S) =              1972/11/20
    UNIVERSAL TIME (H-M-S) =  4-10-20.0
    HORIZ ANGLE (D-M-S) =       60-10-10.0
OUTPUT
    ZENITH DIST (D-M-S) =       45-29-23.2
    GAST (H-M-S) =               8- 7-26.4
    RA (H-M-S) =                 2- 7- 3.7
    DEC (D-M-S) =               89- 8-39.9
    AZ OF STAR (D-M-S) =       359-46-45.9
    AZ OF RO (D-M-S) =         299-36-35.9
```

Appendix C

PROGRAM SPADE LISTING

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                  OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I) NAME(MAIN  ) LINECOUNT(60)

         *....*...1.........2.........3.........4.........5.........6.........7.*.......8

         C***********************************************************************
         C
         C  PROGRAM SPADE2
         C
         C  SOLAR AND POLARIS AZIMUTH DETERMINATION
         C  VERSION 2 - 2 DEC 1983
         C
         C  COMPUTES AZIMUTH OF THE SUN OR POLARIS AND A REFERENCE OBJECT
         C  FROM ASTRONOMICAL OBSERVATIONS ON THE SUN OR POLARIS BY UPDATING
         C  THE CELESTIAL COORDINATES OF THE SUN AND/OR POLARIS.
         C
         C  BY MICHAEL R. CRAYMER
         C     SURVEY SCIENCE
         C     ERINDALE COLLEGE
         C     UNIVERSITY OF TORONTO
         C     L5L 1C6
         C
         C  (C) COPYRIGHT MICHAEL R. CRAYMER, 1983.
         C
         C  INPUT
         C     - FORMAT-FREE (I.E. LEAVE A BLANK SPACE OR COMMA BETWEEN DATA
         C       VALUES.
         C     - DOUBLE PRECISION VALUES ARE INDICATED BY "DP".  EXAMPLES OF
         C       DOUBLE PRECISION VALUES ARE:
         C       30.5D0 -> 30.5
         C       21.D0  -> 21
         C     - START A NEW INPUT RECORD (I.E. LINE OR CARD) FOR EACH NEW
         C       INDIVIDUAL OBSERVATION.  AN INDIVIDUAL OBSERVATION IS CONSIDERED
         C       TO BE THE MEAN VALUES OF ONE INDIVIDUAL OBSERVATION SET.
         C     - CONTINUE INPUT DATA ON THE FOLLOWING INPUT RECORD IF MORE THAN
         C       80 COLUMNS ARE REQUIRED FOR AN INDIVIDUAL OBSERVATION.  BUT
         C       REMEMBER TO BEGIN ON A NEW RECORD FOR A NEW INDIVIDUAL
         C       OBSERVATION.
         C     - THE FOLLOWING DATA VALUES FOR EACH RECORD OR OBSERVATION SHALL
         C       BE ENTERED IN EXACTLY THE SAME ORDER AS GIVEN BELOW!
         C
         C  ORDER OF INPUT
         C     - SOLUTION CODE = 1 -- FOR SOLAR OBSERVATIONS (ZENITH DISTANCE
         C                            SOLUTION)
         C                     = 2 -- FOR SOLAR OBSERVATIONS (HOUR ANGLE SOLUTION
         C                            WITH NO SEMI-DIAMETER CORRECTION)
         C                     = 3 -- FOR SOLAR OBSERVATIONS (HOUR ANGLE SOLUTION
         C                            WITH SEMI-DIAMETER CORRECTION)
         C                     = 4 -- FOR POLARIS OBSERVATIONS (HOUR ANGLE
         C                            SOLUTION)
         C     - DEGREES OF OBSERVER'S LATITUDE (INTEGER)
         C     - MINUTES OF    "          "        "
         C     - SECONDS OF    "          "        "      (DP)
         C     - DEGREES OF OBSERVER'S LONGITUDE (INTEGER)
         C     - MINUTES OF    "          "         "
         C     - SECONDS OF    "          "         "      (DP)
         C     - YEAR (INTERGER)
         C     - MONTH (INTEGER)
```

```
                  *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                  C     - DAY OF OBSERVATION (INTEGER)
                  C     - HOURS OF UNIVERSAL TIME OF OBSERVATION (INTEGER)
                  C     - MINUTES OF  "      " "     "       "
                  C     - SECONDS OF  "      " "     "       "     (DP)
                  C     - DEGREES OF HORIZONTAL ANGLE FROM RO TO SUN OR POLARIS (INTEGER)
                  C     - MINUTES OF    "       "   "  "  "  "  "    "         "
                  C     - SECONDS OF    "       "   "  "  "  "  "    "       .  " (DP)
                  C     - DEGREES OF ZENITH ANGLE (INTEGER) -- IF NOT OBSERVED ENTER 0
                  C     - MINUTES OF  "     "      "        "  "    "        "  "
                  C     - SECONDS OF  "     "      "        "  "    "        "  " (DP)
                  C
                  C  NOTE
                  C     - THE UTC OFFSET FROM ET (DET) IS DEFINED IN THE DATA STATEMENT.
                  C       SHOULD THE UTC OFFSET BE CHANGED, THE DATA STATEMENT SHOULD BE
                  C       MODIFIED ACCORDINGLY FOR HIGHEST PRECISION.
                  C
                  C*************************************************************************
ISN      1              IMPLICIT REAL*8(A-H,O-Z)
ISN      2              REAL*8 JD,JED,MA,LMAN,NL,NOB
ISN      3              INTEGER SOLN,OLATD,OLATM,OLONGD,OLONGM,Y,DAY,UTH,UTM,HAD,HAM,ZDD,
                       &        ZDM,GASTH,GASTM,AZD,AZM,AZROD,AZROM,RAH,RAM,DECD,DECM
                  C-------------------------------------------------------------------------
                  C  INPUT
                  C
ISN      4              DATA RA0/1.8135511D0/,DEC0/89.028817D0/,DRA/0.0050297D0/,
                       &     DDEC/-0.000119D0/,DET/0.015051111/
ISN      5          500 READ(5,*,END=400) SOLN,OLATD,OLATM,OLATS,OLONGD,OLONGM,OLONGS,
                       &     Y,M,DAY,UTH,UTM,UTS,HAD,HAM,HAS,ZDD,ZDM,ZDS
ISN      6              WRITE(6,1001)
                  C-------------------------------------------------------------------------
                  C  COMPUTE REQUIRED DATA
                  C
ISN      7              CALL DEG(OLATD,OLATM,OLATS,OLAT)
ISN      8              CALL DEG(OLONGD,OLONGM,OLONGS,OLONG)
ISN      9              CALL DEG(UTH,UTM,UTS,UT)
ISN     10              CALL DEG(HAD,HAM,HAS,HA)
ISN     11              CALL DEG(ZDD,ZDM,ZDS,ZD)
ISN     12              CALL JDATE(Y,M,DAY,UT,JD)
ISN     13              ET=UT+DET
ISN     14              CALL JDATE(Y,M,DAY,ET,JED)
ISN     15              CALL DATA(JED,MA,LMAN,D,F,NL,NOB,TOB,TL,TR,AL,ALAT)
ISN     16              CALL GST(UT,JD,TOB,NL,GAST)
ISN     17              IF(SOLN.EQ.4)GOTO 100
ISN     18              CALL RADS(AL,ALAT,TOB,RA,DEC)
                  C-------------------------------------------------------------------------
                  C  ZENITH DISTANCE SOLUTION - SOLAR OBSERVATON
                  C
ISN     19              IF(SOLN.NE.1)GOTO 200
ISN     20              CALL AZZD(DEC,UT,OLAT,OLONG,ZD,AZ)
ISN     21              GOTO 300
                  C-------------------------------------------------------------------------
                  C  POLARIS OBSERVATION DATA
                  C
ISN     22          100 CALL PM(JD,RA0,DEC0,DRA,DDEC,RA1,DEC1)
ISN     23              CALL PREC(JD,RA1,DEC1,RA2,DEC2)
ISN     24              CALL NUT(NL,NOB,TOB,RA2,DEC2,RA3,DEC3)
```

```
                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

ISN      25            CALL AAB(TL,TOB,RA3,DEC3,RA4,DEC4)
ISN      26            CALL DAB(OLAT,OLONG,GAST,RA4,DEC4,RA,DEC)
                C-----------------------------------------------------------------------
                C  HOUR ANGLE SOLUTION - SOLAR AND POLARIS OBSERVATION
                C
ISN      27       200 CALL AZHA(RA,DEC,GAST,OLAT,OLONG,ZD,AZ)
                C OPTIONAL SEMI-DIAMETER CORRECTION
ISN      28            IF(SOLN.EQ.3)CALL SDC(HA,ZD,TR,SD)
                C-----------------------------------------------------------------------
                C  OUTPUT
                C
ISN      30       300 AZRO=AZ-HA
ISN      31            IF(AZRO.LT.0.D0)AZRO=AZRO+360
ISN      33            CALL DMS(ZD,ZDD,ZDM,ZDS)
ISN      34            CALL DMS(AZ,AZD,AZM,AZS)
ISN      35            CALL DMS(AZRO,AZROD,AZROM,AZROS)
ISN      36            CALL DMS(GAST,GASTH,GASTM,GASTS)
ISN      37            CALL DMS(RA,RAH,RAM,RAS)
ISN      38            CALL DMS(DEC,DECD,DECM,DECS)
ISN      39            IF(SOLN.EQ.1) WRITE(6,1002)
ISN      41            IF(SOLN.EQ.2) WRITE(6,1003)
ISN      43            IF(SOLN.EQ.3) WRITE(6,1004)
ISN      45            IF(SOLN.EQ.4) WRITE(6,1005)
ISN      47            WRITE(6,1006) OLATD,OLATM,OLATS,OLONGD,OLONGM,OLONGS,Y,M,DAY,
                     &      UTH,UTM,UTS,HAD,HAM,HAS,ZDD,ZDM,ZDS,GASTH,GASTM,GASTS,RAH,
                     &      RAM,RAS,DECD,DECM,DECS,AZD,AZM,AZS,AZROD,AZROM,AZROS
ISN      48            GOTO 500
ISN      49       400 STOP
                C-------------------------------------------------------------------
                C  FORMAT STATEMENTS
                C
ISN      50      1001 FORMAT('1SPADE - SOLAR AND POLARIS AZIMUTH DETERMINATION',//)
ISN      51      1002 FORMAT(1X,'SOLAR OBSERVATION - ZENITH DISTANCE SOLUTION',/)
ISN      52      1003 FORMAT(1X,'SOLAR OBSERVATION - HOUR ANGLE SOLUTION',/,
                     &      1X,'NO SEMI-DIAMETER CORRECTION',/)
ISN      53      1004 FORMAT(1X,'SOLAR OBSERVATION - HOUR ANGLE SOLUTION',/,
                     &      1X,'WITH SEMI-DIAMETER CORRECTION',/)
ISN      54      1005 FORMAT(1X,'POLARIS OBSERVATION',/)
ISN      55      1006 FORMAT(1X,'INPUT',/,5X,
                     &      'LATITUDE (D-M-S) = ',I8,'-',I2,'-',F4.1,/,5X,
                     &      'LONGITUDE (D-M-S) = ',I7,'-',I2,'-',F4.1,/,5X,
                     &      'DATE (Y/M/S) =   ',I12,'/',I2,'/',I2,/,5X,
                     &      'UNIVERSAL TIME (H-M-S) = ',I2,'-',I2,'-',F4.1,/,5X,
                     &      'HORIZ ANGLE (D-M-S) = ',I5,'-',I2,'-',F4.1,/,1X,
                     &      'OUTPUT',/,5X,
                     &      'ZENITH DIST (D-M-S) = ',I5,'-',I2,'-',F4.1,/,5X,
                     &      'GAST (H-M-S) = ',I12,'-',I2,'-',F4.1,/,5X,
                     &      'RA (H-M-S) = ',I14,'-',I2,'-',F4.1,/,5X,
                     &      'DEC (D-M-S) = ',I13,'-',I2,'-',F4.1,/,5X,
                     &      'AZ OF STAR (D-M-S) = ',I6,'-',I2,'-',F4.1,/,5X,
                     &      'AZ OF RO (D-M-S) = ',I8,'-',I2,'-',F4.1,//)
ISN      56             END

*STATISTICS*   SOURCE STATEMENTS = 50, PROGRAM SIZE = 3170 BYTES, PROGRAM NAME = MAIN    PAGE:    1.
```

STATISTICS NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 1 ******

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                   OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I) NAME(MAIN  ) LINECOUNT(60)

             *....*...1.........2.........3.........4.........5.........6.........7.*.......8

             C*************************************************************************
ISN    1           SUBROUTINE AAB(TL,TOB,RA3,DEC3,RA4,DEC4)
             C
             C     ANNUAL CIRCULAR ABERRATION
             C       COMPUTES APPARENT RIGHT ASCENSION AND DECLINATION DISPLACED
             C     BY CIRCULAR ANNUAL ABERRATION GIVEN THE TRUE OBLIQUITY, THE
             C     TRUE RIGHT ASCENSION AND DECLINATION OF THE STAR AND THE TRUE
             C     LONGITUDE OF THE SUN.
             C
             C     R    = CONVERSION FACTOR FROM DEGREES TO RADIANS
             C     TL   = INPUT TRUE LONGITUDE OF THE SUN (DEGS)
             C     TOB  = NPUT TRUE OBLIQUITY (DEGS)
             C     RA3  = INPUT TRUE RIGHT ASCENSION REFERRED TO THE TRUE EQUINOX OF
             C              DATE (HRS)
             C     DEC3 = INPUT TRUE DECLINATION REFERRED TO THE TRUE EQUINOX OF DATE
             C              (DEGS)
             C     RA4  = OUTPUT APPARENT RIGHT ASCENSION DISPLACED BY CIRCULAR ANNUA
             C              ABERRATION (HRS)
             C     DEC4 = OUTPUT APPARENT DECLINATION DISPLACED BY CIRCULAR ANNUAL
             C              ABERRATION (DEGS)
             C*************************************************************************
ISN    2           IMPLICIT REAL*8(A-H,O-Z)
ISN    3           REAL*8 DSIN,DCOS,DTAN,DATAN
ISN    4           R=4.D0*DATAN(1.D0)/180.D0
ISN    5           RA4=RA3-0.0003796D0/DCOS(DEC3*R)*(DCOS(RA3*15*R)*DCOS(TL*R)
                 &   *DCOS(TOB*R)+DSIN(RA3*15*R)*DSIN(TL*R))
ISN    6           DEC4=DEC3-0.005693D0*(DCOS(TL*R)*DCO(TOB*R)*(DTAN(TOB*R)
                 &    *DCOS(DEC3*R)-DSIN(RA3*15*R)*DSIN(DEC3*R))+DCOS(RA3*15*R)
                 &    *DSIN(DEC3*R)*DSIN(TL*R))
ISN    7           RA4=RA3-0.0003796D0/DCOS(DEC4*R)*(DCOS(RA4*15*R)*DCOS(TL*R)
                 &   *DCOS(TOB*R)+DSIN(RA4*15*R)*DSIN(TL*R))
ISN    8           DEC4=DEC3-0.005693D0*(DCOS(TL*R)*DCOS(TOB*R)*(DTAN(TOB*R)
                 &    *DCOS(DEC4*R)-DSIN(RA4*15*R)*DSIN(DEC4*R))+DCOS(RA4*15*R)
                 &    *DSIN(DEC4*R)*DSIN(TL*R))
ISN    9           RETURN
ISN   10           END

*STATISTICS*  SOURCE STATEMENTS = 10, PROGRAM SIZE = 1816 BYTES, PROGRAM NAME = AAB   PAGE:   5.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 2 ******
```

```
LEVEL 1.1.1 (DEC   81)          VS FORTRAN         DATE: JUL 19, 1984    TIME: 03:38:16      NAME: MAIN    PAGE:

OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF   GOSTMT NODECK   SOURCE   TERM   OBJECT FIXED
                     OPTIMIZE(0)  LANGLVL(66) NOFIPS   FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                C**********************************************************************
ISN        1          SUBROUTINE AZHA(RA,DEC,GAST,OLAT,OLONG,ZD,AZ)
                C
                C     AZIMUTH DETERMINAION - HOUR ANGLE SOLUTION
                C       COMPUTES AZTIMUTH AND ZENITH DISTANCE OF A CELESTIAL OBJECT
                C     GIVEN THE GAST, THE OBSERVER'S LATITUDE AND LONGITUDE, THE
                C     RIGHT ASCENSION AND DECLINATION OF THE STAR
                C
                C     R = C  ONVERSION FAC  TOR FROM DEGREES TO RADIANS
                C     GAST = GREENWIC  H APPARENT SIDEREAL TIME (HRS)
                C     RA = APPARENT RIGHT ASC  ENSION (HRS)
                C     DEC   = APPARENT DEC  LINATION (DEGS)
                C     OLAT = OBSERVER'S LATITUDE (DEGS)
                C     OLONG = OBSERVER'S LONGITUDE (DEGS)
                C     H = HOUR ANGLE (DEGS)
                C     AZ = AZIMUTH OF C  ELESTIAL OBJEC  T (DEGS)
                C     ZD = ZENTITH DISTANC  E (DEGS)
                C**********************************************************************
ISN        2          IMPLICIT REAL*8(A-H,O-Z)
ISN        3          REAL*8 DCOS,DSIN,DTAN,DATAN,DARCOS
ISN        4          INTEGER SOLN
ISN        5          H=(GAST-RA)*15.-OLONG
ISN        6          R=(4.*DATAN(1.D0))/180.
ISN        7          V=-DCOS(DEC*R)*DSIN(H*R)
ISN        8          U=DSIN(DEC*R)*DCOS(OLAT*R)-DCOS(DEC*R)*DCOS(H*R)*DSIN(OLAT*R)
ISN        9          AZ=DATAN(V/U)/R
IS        10         IF(U.LT.0.D0)AZ=AZ+180
ISN       12          IF(U.GT.0.D0.AND.V.LT.0.D0)AZ=AZ+360
ISN       14          ZD=DARCOS(DSIN(DEC*R)*DSIN(OLAT*R)+DCOS(DEC*R)*DCOS(H*R)
                     &  *DCOS(OLAT*R))/R
ISN       15          RETURN
ISN       16          END

*STATISTICS*   SOURCE STATEMENTS = 14, PROGRAM SIZE = 1198 BYTES, PROGRAM NAME = AZHA    PAGE:   6.

*STATISTICS*    NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 3 ******
```

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                    OPTIMIZE(0)  LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

              *....*...1.........2.........3.........4.........5.........6.........7.*.......8

              C**********************************************************************
ISN      1          SUBROUTINE AZZD(DEC,UT,OLAT,OLONG,ZD,AZ)
              C
              C     AZIMUTH DETERMINATION  ZENITH DISTANCE SOLUTION
              C       COMPUTES AZIMUTH OF CELESTIAL OBJECT GIVEN THE
              C     OBSERVED ZENITH DISTANCE, APPROX. UT, OBSERVER'S LATITUDE AND
              C     LONGITUDE
              C
              C     R = C  ONVERSION FAC  TOR FROM DEGREES TO RADIANS
              C     ZD = ZENITH DISTANC  E (DEGS) (INPUT IS OBSERVED; OUTPUT IS
              C          C  ORREC  TED FOR REFRAC  TION AND GEOC  ENTRIC   PARALLAX)
              C     OLAT = OBSERVER'S LATITUDE (DEGS)
              C     OLONG = OBSERVER'S LONGITUDE (DEGS)
              C     DEC   = APPARENT DEC  LINATION (DEGS)
              C     UT = APPROX. UNIVERSAL TIME (HRS)
              C     AZ = AZIMUTH OF C  ELESTIAL OBJEC  T (DEGS)
              C**********************************************************************
ISN      2          IMPLICIT REAL*8(A-H,O-Z)
ISN      3          REAL*8 DSIN,DCOS,DTAN,DATAN,DARCOS
ISN      4          R=(4.*DATAN(1.D0))/180.
ISN      5          ZD=ZD+(58*DTAN(ZD*R)-8.9*DSIN(ZD*R))/3600.
ISN      6          AZ=DARCOS((DSIN(DEC*R)-DSIN(OLAT*R)*DCOS(ZD*R))/(DCOS(OLAT*R)
                  %   *DSIN(ZD*R)))/R
ISN      7          IF(AZ.LT.0.D0.AND.(UT-OLONG/15.).LT.12.)AZ=AZ+180.
ISN      9          IF(AZ.LT.0.D0.AND.(UT-OLONG/15.).GT.12.)AZ=180.-AZ
ISN     11          IF(AZ.GE.0.D0.AND.(UT-OLONG/15.).GT.12.)AZ=360.-AZ
ISN     13          RETURN
ISN     14          END

*STATISTICS*  SOURCE STATEMENTS = 11, PROGRAM SIZE = 1038 BYTES, PROGRAM NAME = AZZD   PAGE:   7.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 4 ******
```

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF GOSTMT NODECK SOURCE TERM OBJECT FIXED
                   OPTIMIZE(0) LANGLVL(66) NOFIPS FLAG(I) NAME(MAIN ) LINECOUNT(60)

               *....*...1.........2.........3.........4.........5.........6.........7.*.......8

               C***********************************************************************
ISN      1           SUBROUTINE DAB(OLAT,OLONG,GAST,RA4,DEC4,RA5,DEC5)
               C
               C     DIURNAL ABERRATION
               C       COMPUTES RIGHT ASCENSION AND DECLINATION DISPLACED BY DIURNAL
               C     ABERRATION GIVEN THE UNAFFECTED RIGHT ASCENSION AND DECLINATION,
               C     OBSERVERS LATITUDE AND LONGITUDE AND GREENWICH APPARENT SIDEREAL
               C     TIME.
               C
               C     R     = CONVERSION FACTOR FROM DEGREES TO RADIANS
               C     H1    = APPROXIMATE HOUR ANGLE (DEGS)
               C     GAST  = INPUT GREENWICH APPARENT SIDEREAL TIME (HRS)
               C     OLAT  = INPUT OBSERVERS LATITUDE (DEGS)
               C     OLONG = INPUT OBSERVERS LONGITUDE (DEGS)
               C     RA4   = INPUT RIGHT ASCENSION UNAFFECTED BY DIURNAL ABERRATION (HR
               C     DEC4  = INPUT DECLINATION UNAFFECTED BY DIURNAL ABERRATION (DEGS)
               C     RA5   = OUTPUT RIGHT ASCENSION DISPLACED BY DIURNAL ABERRATION (HR
               C     DEC5  = OUTPUT DECLINATION DISPLACED BY DIURNAL ABERRATION (DEGS)
               C***********************************************************************
ISN      2           IMPLICIT REAL*8(A-H,O-Z)
ISN      3           REAL*8 DSIN,DCOS,DATAN
ISN      4           R=4.D0*DATAN(1.D0)/180.D0
ISN      5           H1=(GAST-RA4)*15-OLONG
ISN      6           RA5=RA4+0.000089D0*DCOS(OLAT*R)*DCOS(H1*R)/DCOS(DEC4*R)/15
ISN      7           DEC5=DEC4+0.000089D0*DCOS(OLAT*R)*DSIN(H1*R)*DSIN(DEC4*R)
ISN      8           RA5=RA4+0.000089D0*DCOS(OLAT*R)*DCOS(H1*R)/DCOS(DEC5*R)/15
ISN      9           H1=(GAST-RA5)*15-OLONG
ISN     10           DEC5=DEC4+0.000089D0*DCOS(OLAT*R)*DSIN(H1*R)*DSIN(DEC5*R)
ISN     11           RETURN
ISN    @12           END

*STATISTICS*  SOURCE STATEMENTS = 12, PROGRAM SIZE = 1086 BYTES, PROGRAM NAME = DAB   PAGE:    8.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 5 ******
```

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF   GOSTMT NODECK   SOURCE   TERM   OBJECT FIXED
                     OPTIMIZE(0)  LANGLVL(66) NOFIPS   FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                C*************************************************************************
ISN      1            SUBROUTINE DATA(JD,MA,LMAN,D,F,NL,NOB,TOB,TL,TR,AL,ALAT)
                C
                C     ASTRONOMICAL DATA
                C       COMPUTES - PERTURBATIONS IN ECLIPTIC LONGITUDE AND LATITUDE AND
                C                  LOG OF RADIUS VECTOR OF SUN
                C                - ANNUAL ABERRATION CORRECTION TO ECLIPTIC LONGITUDE
                C                  OF SUN
                C                - NUTATION IN ECLIPTIC LONGITUDE AND OBLIQUITY
                C                - APPARENT ECLIPTIC LONGITUDE AND LATITUDE AND TRUE
                C                  TRUE LOG OF RADIUS VECTOR OF SUN AND OBLIQUITY
                C       GIVEN THE JULIAN EPHEMERIS DATE OF EPOCH.
                C
                C     JD   = INPUT JULIAN EPHEMERIS DATE (DAYS)
                C     T    = INTERVAL OF EPHEMERIS CENTURIES ELAPSED SINCE 1900 JAN 0.5
                C     MA   = OUTPUT SUN'S MEAN ANOMALY (DEGS)
                C     LMAN = OUTPUT LONGITUDE OF MOON'S MEAN ASCENDING NODE (DEGS)
                C     D    = OUTPUT MEAN ELONGATION OF MOON FROM THE SUN (DEGS)
                C     F    = OUTPUT MOON'S MEAN ARGUMENT OF LATITUDE (DEGS)
                C     NL   = OUTPUT NUTATION IN LONGITUDE (DEGS)
                C     NOB  = OUTPUT NUTATION IN OBLIQUITY (DEGS)
                C     TOB  = OUTPUT TRUE OBLIQUITY (DEGS)
                C     TL   = OUTPUT TRUE LONGITUDE OF SUN (DEGS)
                C     TR   = OUTPUT TRUE DISTANCE FROM THE SUN (AU)
                C     AL   = OUTPUT APPARENT LONGITUDE OF SUN (DEGS)
                C     ALAT = OUTPUT APPARENT LATITUDE OF SUN (DEGS)
                C*************************************************************************
ISN      2            IMPLICIT REAL*8(A-H,O-Z)
ISN      3            REAL*8 JD,LMAN,MA,MAV,MAM,MAJ,MAS,MAMN,ML,MLR,MOB,NL,NOB,
                     &      DSIN,DCOS,DTAN,DARSIN,DATAN
ISN      4            R=4.D0*DATAN(1.D0)/180.D0
ISN      5            T=(JD-2415020.D0)/36525.D0
ISN      6            ML=279.69668D0+36000.76893D0*T+0.00030D0*T*T
ISN      7            MA=358.47583D0+35999.04975D0*T-.00015D0*T*T
ISN      8            C=(1.91946D0-0.00479D0*T-0.00001D0*T*T)*DSIN(MA*R)
                     & +(0.02009D0-0.0001D0*T)*DSIN(2*MA*R)
                     & +0.00029D0*DSIN(3*MA*R)+0.000005D0*DSIN(4*MA*R)
ISN      9            MLR=0.0000306D0-0.0000002D0*T
                     &   +(-0.0072741D0+0.0000181D0*T)*DCOS(MA*R)
                     &   +(-0.0000914D0+0.0000005D0*T)*DCOS(2*MA*R)
                     &   -0.0000015D0*DCOS(3*MA*R)
ISN     10            MOB=23.452294D0-0.013013D0*T
                C
                C  COMPUTE PERTURBATIONS IN ECLIPTIC LONGITU, LITUDE AND LOGR
                C
ISN     11            MAV=212.60322D0+58517.80388D0*T+0.00129D0*T*T
ISN     12            MAM=319.52902D0+19139.85922D0*T+0.00018D0*T*T
ISN     13            MAJ=225.32833D0+3034.96202D0*T-0.00072D0*T*T
ISN     14            MAS=175.46622D0+1221.55147D0*T-0.0005D0*T*T
ISN     15            MAMN=296.10471D0+477198.84911D0*T+0.00919D0*T*T
ISN     16            D=350.73749D0+445267.11422D0*T-0.00144D0*T*T
ISN     17            F=11.25089D0+483202.02515D0*T-.00321D0*T*T
ISN     18            PLV=4.838*DCOS((299.10167D0+MAV-MA)*R)
```

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               *....*...1.........2.........3.........4.........5.........6.........7.*.......8
                   &  +0.116*DCOS((148.9+2*MAV-MA)*R)
                   &  +5.526*DCOS((148.31333D0+2*MAV-2*MA)*R)
                   &  +2.497*DCOS((315.943333D0+2*MAV-3*MA)*R)
                   &  +0.666*DCOS((177.71+3*MAV-3*MA)*R)
                   &  +1.559*DCOS((345.25333D0+3*MAV-4*MA)*R)
                   &  +1.024*DCOS((318.15+3*MAV-5*MA)*R)
                   &  +0.21*DCOS((206.2+4*MAV-4*MA)*R)
                   &  +0.144*DCOS((195.4+4*MAV-5*MA)*R)
                   &  +0.152*DCOS((343.8+4*MAV-6*MA)*R)
                   &  +0.123*DCOS((195.3+5*MAV-7*MA)*R)
                   &  +0.154*DCOS((359.6+5*MAV-8*MA)*R)
ISN      19          PLM=0.273*DCOS((217.7-MAM+MA)*R)
                   &  +2.043*DCOS((343.88833D0-2*MAM+2*MA)*R)
                   &  +1.77*DCOS((200.40167-2*MAM+MA)*R)
                   &  +0.129*DCOS((294.2-3*MAM+3*MA)*R)
                   &  +0.425*DCOS((338.88-3*MAM+2*MA)*R)
                   &  +0.5*DCOS((105.18-4*MAM+3*MA)*R)
                   &  +0.585*DCOS((334.06-4*MAM+2*MA)*R)
                   &  +0.204*DCOS((100.8-5*MAM+3*MA)*R)
                   &  +0.154*DCOS((227.4-6*MAM+4*MA)*R)
I-       20        PLJ=0.163*DCOS((198.6-MAJ+2*MA)*R)
                   &  +7.208*DCOS((179.53167D0-MAJ+MA)*R)
                   &  +2.6*DCOS((263.21667D0-MAJ)*R)
                   &  +2.731*DCOS((87.1450-2*MAJ+2*MA)*R)
                   &  +1.61*DCOS((109.49333D0-2*MAJ+MA)*R)
                   &  +0.164*DCOS((170.5-3*MAJ+3*MA)*R)
                   &  +0.556*DCOS((82.65-3*MAJ+2*MA)*R)
                   &  +0.21*DCOS((98.5-3*MAJ+MA)*R)
ISN      21          PLS=0.419*DCOS((100.58-MAS+MA)*R)
                   &   +0.32*DCOS((269.46-MAS)*R)
                   &  +0.108*DCOS((290.6-2*MAS+2*MA)*R)
                   &  +0.112*DCOS((293.6-2*MAS+MA)*R)
ISN      22          PLMN=6.454*DSIN(D*R)+0.177*DSIN((D+MAMN)*R)
                   &  -0.424*DSIN((D-MAMN)*R)+0.172*DSIN((D-MA)*R)
ISN      23          PLP=6.40*DSIN((231.19+20.30*T)*R)
                   &   +0.27*DSIN((31.8+119*T)*R)
                   &   +(1.88-0.02*T)*DSIN((57.24+150.27*T)*R)
                   &   +0.20*DSIN((315.6+893.3*T)*R)
ISN      24          PL=(PLV+PLM+PLJ+PLS+PLMN+PLP)/3600.
ISN      25          PLR=(236*DCOS((209.08+MAV-MA)*R)
                   &   +684*DCOS((58.31833D0+2*MAV-2*MA)*R)
                   &   +105*DCOS((87.57+3*MAV-3*MA)*R)
                   &   +150*DCOS((225.25+3*MAV-4*MA)*R)
                   &   +206*DCOS((253.82833D0-2*MAM+2*MA)*R)
                   &   +707*DCOS((89.545-MAJ+MA)*R)+133*DCOS(D*R))*1.D-8
ISN      26          PLAT=(0.092*DCOS((93.7+MAV-2*MA)*R)
                   &     +0.21*DCOS((151.8+3*MAV-4*MA)*R)
                   &     +0.166*DCOS((265.5-2*MAJ+MA)*R)
                   &     +0.567*DSIN(F*R)-0.047*DSIN((F-MAMN)*R)
                   &     +0.067*DCOS((123+2*MAV-3*MA)*R))/3600.
               C
               C  COMPUTE NUTATION IN ECLIPTIC LONGITUDE AND OBLIQUITY
               C
ISN      27          LMAN=259.18328D0-1934.14201D0*T+0.00208D0*T*T
ISN      28          NL=(-17.233*DSIN(LMAN*R)+0.209*DSIN(LMAN*2*R)
                  &  -1.273*DSIN((2*LMAN-2*D+2*F)*R)
```

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              *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                 &  +0.126*DSIN(MA*R)-0.204*DSIN((2*LMAN+2*F)*R))/3600
ISN     29        NOB=(9.21*DCOS(LMAN*R)-0.09*DCOS(2*LMAN*R)
                 &   +0.552*DCOS((2*LMAN-2*D+2*F)*R))/3600.D0
              C
              C  COMPUTE ANNUAL ABBERATION CORRECTION TO ECLIPTIC LONGITUDE
              C
ISN     30        E=0.01675104D0-0.0000418D0*T-0.000000126D0*T*T
ISN     31        W=101.220833D0+1.71918D0*T+0.00045D0*T*T
ISN     32        ABL=20.496*(E*DCOS((W-ML-C-PL-NL)*R)-1)/3600./(10.D0**R)
              C
              C  COMPUTE APPARENT ECLIPTIC LONGITUDE, LATITUDE, TRUE RADIUS VECTOR
              C  AND TRUE OBLIQUITY.
              C
ISN     33        TL=ML+C+PL+NL
ISN     34        AL=TL+ABL
ISN     35        AL=AL-IDINT(AL/360.D0)*360.D0
ISN     36        ALAT=PLAT
ISN     37        TLR=MLR+PLR
ISN     38        TR=10.D0**TLR
ISN     39        TOB=MOB+NOB
ISN     40        RETURN
ISN     41        END

*STATISTICS*   SOURCE STATEMENTS = 41, PROGRAM SIZE = 7644 BYTES, PROGRAM NAME = DATA    PAGE:    9.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 6 ******
```

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                    OPTIMIZE(0)  LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

              *....*...1.........2.........3.........4.........5.........6.........7.*.......8

              C************************************************************************
ISN      1          SUBROUTINE DEG(D,M,S,DD)
              C
              C     DEGREES-MINUTES-SECONDS TO DECIMAL DEGREES CONVERSION
              C       CONVERTS ANGLE IN DEGREES (OR HOURS), MINUTES AND SECONDS TO
              C     AN ANGLE IN DECIMAL DEGREES (OR HOURS).
              C
              C     D  = INPUT DEGREES (OR HOURS) - INTEGER
              C     M  = INPUT MINUTES - INTEGER
              C     S  = INPUT SECONDS - DOUBLE PRECISION
              C     DD = OUTPUT DECIMAL DEGREES (OR HOURS) - DOUBLE PRECISION
              C************************************************************************
ISN      2          REAL*8 DD,S
ISN      3          INTEGER D,M
ISN      4          DD=IABS(D)+M/60.D0+S/3600.D0
ISN      5          IF(D.LT.0.D0)DD=-DD
ISN      7          RETURN
ISN      8          END

*STATISTICS*  SOURCE STATEMENTS = 7, PROGRAM SIZE = 440 BYTES, PROGRAM NAME = DEG   PAGE:   12.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 7 ******
```

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                    OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

              *....*...1.........2.........3.........4.........5.........6.........7.*.......8

              C***********************************************************************
ISN      1          SUBROUTINE DMS(DD,D,M,S)
              C
              C     DECIMAL DEGREES TO DEGREES-MINUTES-SECONDS CONVERSION
              C       CONVERTS ANGLE IN DECIMAL DEGREES (OR HOURS) TO AN ANGLE IN
              C     IN DEGREES (OR @OURS), MINUTES AND SECONDS.
              C
              C     DD = INPUT DECIMAL DEGREES (OR HOURS) - DOUBLE PRECISION
              C     D  = OUTPUT DEGREES (OR HOURS) - INTEGER
              C     M  = OUTPUT MINUTES - INTEGER
              C     S  = OUTPUT SECONDS - DOUBLE PRECISION
              C***********************************************************************
ISN      2          REAL*8 DD,S,DABS
ISN      3          INTEGER D,M
ISN      4          D=IDINT(DD)
ISN      5          M=IDINT(DABS(DD-D)*60.D0)
ISN      6          S=(DABS(DD-D)*60.D0-M)*60.D0
ISN      7          RETURN
ISN      8          END

*STATISTICS*   SOURCE STATEMENTS = 8, PROGRAM SIZE = 472 BYTES, PROGRAM NAME = DMS    PAGE:   13.

*STATISTICS*    NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 8 ******
```

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                   OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

          *....*...1.........2.........3.........4.........5.........6.........7.*.......8

          C***********************************************************************
ISN    1        SUBROUTINE GST(UT,JD,TOB,NL,GAST)
          C
          C     GREENWICH APPARENT SIDEREAL TIME
          C       COMPUTES GREENWICH APPARENT SIDEREAL TIME GIVEN THE UNIVERSAL
          C     TIME, JULIAN DATE, TRUE OBLIQUITY AND NUTATION IN LONGITUDE OF
          C     DATE
          C
          C     R    = CONVERSION FACTOR FROM DEGREES TO RADIANS
          C     UT   = INPUT UNIVERSAL TIME (HRS)
          C     JD   = INPUT JULIAN EPHEMERIS DATE (DAYS)
          C     NL   = INPUT NUTATION IN LONGITUDE (DEGS)
          C     TOB  = INPUT TRUE OBLIQUITY (DEGS)
          C     GMST = GREENWICH MEAN SIDEREAL TIME (HRS)
          C     GAST = OUTPUT GREENWICH APPARENT SIDEREAL TIME (HRS)
          C***********************************************************************
ISN    2        IMPLICIT REAL*8(A-H,O-Z)
ISN    3        REAL*8 JD,NL,DCOS,DATAN
ISN    4        R=4.D0*DATAN(1.D0)/180.D0
ISN    5        T=(JD-2415020.D0)/36525.D0
ISN    6        GMST=UT+6.646065D0+2400.051262D0*T+0.000026D0*T*T
ISN    7        GAST=GMST+NL*DCOS(TOB*R)/15.D0
ISN    8        GAST=GAST-IDINT(GAST/24.D0)*24.D0
ISN    9        RETURN
ISN   10        END

*STATISTICS*  SOURCE STATEMENTS = 10, PROGRAM SIZE = 640 BYTES, PROGRAM NAME = GST   PAGE:  14.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 9 ******
```

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                      OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                C*************************************************************************
ISN        1          SUBROUTINE JDATE(Y,M,D,UT,JD)
                C
                C     JULIAN DATE
                C       COMPUTES JULIAN DATE GIVEN THE TIME, DAY, MONTH AND YEAR
                C
                C     Y  = INPUT YEAR - INTEGER
                C     M  = INPUT MONTH - INTEGER
                C     D  = INPUT DAY - INTEGER
                C     UT = INPUT UNIVERSAL TIME (HRS)
                C     DD = DAY INCLUDING FRACTIONAL PART - DOUBLE PRECISION
                C     JD = OUTPUT JULIAN DATE (DAYS)
                C*************************************************************************
ISN        2          IMPLICIT REAL*8(A-H,O-Z)
ISN        3          REAL*8 JD
ISN        4          INTEGER Y,M,D,YY,MM,A,B
ISN        5          YY=Y
ISN        6          MM=M
ISN        7          IF(M.LE.2)YY=Y-1
ISN        9          IF(M.LE.2)MM=M+12
ISN       11          A=IDINT(YY/100.D0)
ISN       12          B=2.D0-A+IDINT(A/4.D0)
ISN       13          DD=D+UT/24.D0
ISN       14          JD=IDINT(365.25D0*YY)+IDINT(30.6001D0*(MM+1))+DD+1720994.5D0+B
ISN       15          RETURN
ISN       16          END

*STATISTICS*  SOURCE STATEMENTS = 14, PROGRAM SIZE = 852 BYTES, PROGRAM NAME = JDATE   PAGE:  15.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 10 ******
```

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                    OPTIMIZE(0)  LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

            *....*...1.........2.........3.........4.........5.........6.........7.*.......8

            C************************************************************************
ISN    1          SUBROUTINE NUT(NL,NOB,TOB,RA2,DEC2,RA3,DEC3)
            C
            C     NUTATION
            C       COMPUTES RIGHT ASCENSION AND DECLINATION REFERRED TO THE
            C     TRUE ECLIPTIC OF DATE FROM THE RIGHT ASCENSION AND
            C     DECLINATION REFERRED TO THE MEAN ECLIPTIC OF DATE GIVEN
            C     MA,LMAN,D,F,NL,TOB.
            C
            C     R    = CONVERSION FACTOR FROM DEGREES TO RADIANS
            C     NL   = INPUT NUTATION IN LONGITUDE (DEGS)
            C     NOB  = INPUT NUTATION IN OBLIQUITY (DEGS)
            C     TOB  = INPUT TRUE OBLIQUITY OF DATE (DEGS)
            C     RA2  = INPUT RIGHT ASCENSION REFERRED TO MEAN EQUINOX OF DATE (HRS
            C     DEC2 = INPUT DECLINATION REFERRED TO MEAN EQUINOX OF DATE (DEGS)
            C     RA3  = OUTPUT RIGHT ASCENSION REFERRED TO TRUE EQUINOX OF DATE (HR
            C     DEC3 = OUTPUT DECLINATION REFERRED TO TRUE EQUINOX OF DATE (DEGS)
            C************************************************************************
ISN    2          IMPLICIT REAL*8(A-H,O-Z)
ISN    3          REAL*8 NL,NOB,DSIN,DCOS,DTAN,DARSIN,DATAN,
                 &       N11,N12,N13,N21,N22,N23,N31,N32,N33
ISN    4          R=4.D0*DATAN(1.D0)/180.D0
ISN    5          A=TOB+NOB
ISN    6          N11=DCOS(NL*R)
ISN    7          N12=-1*DSIN(NL*R)*DCOS(TOB*R)
ISN    8          N13=-1*DSIN(NL*R)*DSIN(TOB*R)
ISN    9          N21=DCOS(A*R)*DSIN(NL*R)
ISN   10          N22=DCOS(A*R)*DCOS(NL*R)*DCOS(TOB*R)+DSIN(A*R)*DSIN(TOB*R)
ISN   11          N23=DCOS(A*R)*DCOS(NL*R)*DSIN(TOB*R)-DSIN(A*R)*DCOS(TOB*R)
ISN   12          N31=DSIN(A*R)*DSIN(NL*R)
ISN   13          N32=DSIN(A*R)@DCOS(NL*R)*DCOS(TOB*R)-DCOS(A*R)*DSIN(TOB*R)
ISN   14          N33=DSIN(A*R)*DCOS(NL*R)*DSIN(TOB*R)+DCOS(A*R)*DCOS(TOB*R)
ISN   15          X2=DCOS(DEC2*R)*DCOS(RA2*15*R)
ISN   16          Y2=DCOS(DEC2*R)*DSIN(RA2*15*R)
ISN   17          Z2=DSIN(DEC2*R)
ISN   18          X3=N11*X2+N12*Y2+N13*Z2
ISN   19          Y3=N21*X2+N22*Y2+N23*Z2
ISN   20          Z3=N31*X2+N32*Y2+N33*Z2
ISN   21          RA3=DATAN(Y3/X3)/15/R
ISN   22          IF(X3.LT.0.D0) RA3=RA3+12.D0
ISN   24          IF(X3.GE.0.D0.AND.Y3.LT.0.D0) RA3=RA3+24.D0
ISN   26          DEC3=DARSIN(Z3)/R
ISN   27          RETURN
ISN   28          END

*STATISTICS*  SOURCE STATEMENTS = 26, PROGRAM SIZE = 2374 BYTES, PROGRAM NAME = NUT   PAGE:  16.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 11 ******
```

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF GOSTMT NODECK SOURCE TERM OBJECT FIXED
                   OPTIMIZE(0) LANGLVL(66) NOFIPS FLAG(I) NAME(MAIN ) LINECOUNT(60)

                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                C**************************************************************************
ISN      1            SUBROUTINE PM(JD,RA0,DEC0,DRA,DDEC,RA1,DEC1)
                C
                C     PROPER MOTION
                C       COMPUTES RIGHT ASCENSION AND DECLINATION OF CURRENT EPOCH
                C     AND CATALOG EQUINOX GIVEN THE RIGHT ASCENSION AND
                C     DECLINATION OF THE CATALOG EPOCH AND EQUINOX, CENTENIAL PROPER
                C     MOTIONS IN RA AND DEC, AND THE JULIAN DATE.
                C
                C     RA0  = INPUT CATALOGED RIGHT ASCENSION FOR 1950.0 (HRS)
                C     DEC0 = INPUT CATALOGED DECLINATION FOR 1950.0 (DEG)
                C     JD   = INPUT JULIAN DATE (DAYS)
                C     DRA  = INPUT CENTENIAL PROPER MOTION IN RIGHT ASCENSION (HRS)
                C     DDEC = INPUT CENTENIAL PROPER MOTION IN DECLINATION (DEGS)
                C     TO   = INTERVAL OF TPIC CENTURIES ELAPSED SINCE 1950.0
                C     RA1  = OUTPUT RIGHT ASCENSION REFERRED TO CURRENT EPOCH AND
                C              CATALOGED EQUINOX (HRS)
                C     DEC1 = OUTPUT DECLINATION REFERRED TO CURRENT EPOCH AND CATALOGED
                C              EQUINOX (DEG)
                C**************************************************************************
ISN      2            IMPLICIT REAL*8(A-H,O-Z)
ISN      3            REAL*8 JD
ISN      4            TO=(JD-2433282.423D0)/36524.2199D0
ISN      5            RA1=RA0+DRA*TO
ISN      6            DEC1=DEC0+DDEC*TO
ISN      7            RETURN
ISN      8            END

*STATISTICS*  SOURCE STATEMENTS = 8, PROGRAM SIZE = 382 BYTES, PROGRAM NAME = PM    PAGE:   17.

*STATISTICS*  NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 12 ******
```

```
OPTIONS IN EFFECT: NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                   OPTIMIZE(0)  LANGLVL(66) NOFIPS   FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

            *....*...1.........2.........3.........4.........5.........6.........7.*.......8

            C*************************************************************************
ISN     1         SUBROUTINE PREC(JD,RA1,DEC1,RA2,DEC2)
            C
            C     PRECESSION
            C       COMPUTES THE MEAN RIGHT ASCENSION AND DECLINATION OF A STAR
            C     GIVEN THE JULIAN DATE, THE RIGHT ASCENSION
            C     AND DECLINATION FOR THE EPOCH 1950.0 CORRECTED FOR PROPER
            C     MOTION.
            C
            C     JD   = INPUT JULIAN EPHEMERIS DATE OF CURRENT EPOCH (DAYS)
            C     TO   = INTERVAL OF TROPICAL CENTURIES ELAPSED SINCE 1950.0
            C            (JD=2433282.423)
            C     RA1  = INPUT RIGHT ASCENSION OF 1950.0 EQUINOX CORRECTED FOR PROPE
            C            MOTION (HRS)
            C     DEC1 = INPUT DECLINATION OF 1950.0 EQUINOX CORRECTED FOR PROPER
            C            MOTION (DEG)
            C     RA2  = OUTPUT MEAN RIGHT ASCENSION FOR CURRENT EPOCH (HRS)
            C     DEC2 = OUTPUT MEAN DECLINATION FOR CURRENT EPOCH (DEGS)
            C*************************************************************************
ISN     2         IMPLICIT REAL*8(A-H,O-Z)
ISN     3         REAL*8 JD,DSIN,DCOS,DTAN,DARSIN,DATAN
ISN     4         R=4.D0*DATAN(1.D0)/180.D0
ISN     5         TO=(JD-2433282.423D0)/36524.2199D0
ISN     6         ZETA=0.6402633D0*TO+0.0000839*TO**2+0.000005*TO**3
ISN     7         Z=ZETA+0.0002197*TO**2
ISN     8         THETA=0.5567376*TO-0.0001183*TO**2+0.0000117*TO**3
ISN     9         P11=DCOS(Z*R)*DCOS(THETA*R)*DCOS(ZETA*R)-DSIN(Z*R)*DSIN(ZETA*R)
ISN    10         P12=-1*DCOS(Z*R)*DCOS(THETA*R)*DSIN(ZETA*R)
                 &    -DSIN(Z*R)*DCOS(ZETA*R)
ISN    11         P13=-1*DCOS(Z*R)*DSIN(THETA*R)
ISN    12         P21=DSIN(Z*R)*DCOS(THETA*R)*DCOS(ZETA*R)+DCOS(Z*R)*DSIN(ZETA*R)
ISN    13         P22=-1*DSIN(Z*R)*DCOS(THETA*R)*DSIN(ZETA*R)
                 &    +DCOS(Z*R)*DCOS(ZETA*R)
ISN    14         P23=-1*DSIN(Z*R)*DSIN(THETA*R)
ISN    15         P31=DSIN(THETA*R)*DCOS(ZETA*R)
ISN    16         P32=-1*DSIN(THETA*R)*DSIN(ZETA*R)
ISN    17         P33=DCOS(THETA*R)
ISN    18         X1=DCOS(DEC1*R)*DCOS(RA1*15*R)
ISN    19         Y1=DCOS(DEC1*R)*DSIN(RA1*15*R)
ISN    20         Z1=DSIN(DEC1*R)
ISN    21         X2=P11*X1+P12*Y1+P13*Z1
ISN    22         Y2=P21*X1+P22*Y1+P23*Z1
ISN    23         Z2=P31*X1+P32*Y1+P33*Z1
ISN    24         RA2=DATAN(Y2/X2)/15/R
ISN    25         IF(X2.LT.0.D0) RA2=RA2+12.D0
ISN    27         IF(X2.GE.0.D0.AND.Y2.LT.0.D0) RA2=RA2+24.D0
ISN    29         DEC2=DARSIN(Z2)/R
ISN    30         RETURN
ISN    31         END

*STATISTICS*  SOURCE STATEMENTS = 29, PROGRAM SIZE = 2626 BYTES, PROGRAM NAME = PREC   PAGE:  18.

*STATISTICS*  NO DIAGNOSTICS GENERATED.
```

****** END OF COMPILATION 13 ******

```
OPTIONS IN EFFECT:  NOLIST NOMAP NOXREF  GOSTMT NODECK  SOURCE  TERM  OBJECT FIXED
                    OPTIMIZE(0) LANGLVL(66) NOFIPS  FLAG(I)  NAME(MAIN  )  LINECOUNT(60)

            *....*...1.........2.........3.........4.........5.........6.........7.*......8

            C***********************************************************************
ISN     1         SUBROUTINE RADS(AL,ALAT,TOB,RA,DEC)
            C
            C     RIGHT ASCENSION & DECLINATION OF THE SUN
            C       COMPUTES APPARENT GEOCENTRIC RIGHT ASCENSION AND DECLINATION
            C     GIVEN THE APPARENT GEOCENTRICLATITUDE AND LONGITUDE OF THE
            C     SUN AND THE TRUE OBLIQUITY OF THE ECLIPTIC.
            C
            C     R = C  ONVERSION FAC  TOR FROM DEGREES TO RADIANS
            C     AL = APPARENT GEOC  ENTRIC   LONGITUDE OF THE SUN (DEGS)
            C     ALAT - APPARENT GEOC  ENTRIC   LATITUDE OF THE SUN (DEGS)
            C     TOB = TRUE OBLIQUITY (DEGS)
            C     RA = APPARENT RIGHT ASC  ENSON OF THE SUN (HRS)
            C     DEC   = APPARENT DEC  LINATION (DEGS)
            C***********************************************************************
ISN     2         IMPLICIT REAL*8(A-H,O-Z)
ISN     3         REAL*8 DSIN,DCOS,DARSIN,DATAN
ISN     4         R=4*DATAN(1.D0)/180
ISN     5         V=DCOS(ALAT*R)*DCOS(AL*R)
ISN     6         U=DCOS(ALAT*R)*DSIN(AL*R)*DCOS(TOB*R)-DSIN(ALAT*R)*DSIN(TOB*R)
ISN     7         RA=DATAN(U/V)/R/15
ISN     8         IF(V.LT.0.D0)RA=RA+12
ISN    10         IF(V.GT.0.D0.AND.U.LT.0.D0)RA=RA+24
ISN    12         DEC=DARSIN(DCOS(ALAT*R)*DSIN(AL*R)*DSIN(TOB*R)+DSIN(ALAT*R)
                 &    *DCOS(TOB*R))/R
ISN    13         RETURN
ISN    14         END

*STATISTICS*  SOURCE STATEMENTS = 12, PROGRAM SIZE = 1128 BYTES, PROGRAM NAME = RADS    PAGE:   20.

*STATISTICS*   NO DIAGNOSTICS GENERATED.

****** END OF COMPILATION 14 ******
```

OPTIONS IN EFFECT: NOLIST NOMAP NOXREF GOSTMT NODECK SOURCE TERM OBJECT FIXED
OPTIMIZE(0) LANGLVL(66) NOFIPS FLAG(I) NAME(MAIN) LINECOUNT(60)

```
                *....*...1.........2.........3.........4.........5.........6.........7.*.......8

                C***************************************************************************
ISN       1           SUBROUTINE SDC(HA,ZD,TR,SD)
                C
                C     SEMI-DIAMETER CORRECTION
                C       COMPUTES THE SEMI-DIAMETER CORRECTION AND APPLIES IT TO THE
                C     OBSERVED HORIZONTAL ANGLE MEASURED CLOCKWISE FROM THE RO TO THE
                C     SUN GIVEN THE TRUE DISTANCE TO THE SUN, THE OBSERVED ZENITH
                C     ANGLE AND THE OBSERVED HORIZONTAL ANGLE.
                C
                C     TR = TRUE RADIUS VEC  TOR (AU)
                C     ZD = ZENITH DISTANC  E (DEGS)
                C     SD = SEMI-DIAMETER (DEGS)
                C     HA = HORIZONTAL ANGLE (DEGS)
                C***************************************************************************
ISN       2           IMPLICIT REAL*8(A-H,O-Z)
ISN       3           REAL*8 DSIN,DATAN
ISN       4           R=4*DATAN(1.D0)/180
ISN       5           SD=0.266994/TR
ISN       6           HA=SD/DSIN(ZD*R)+HA
ISN       7           RETURN
ISN       8           END
```